National Technical University of Athens (NTUA)

Department of Civil Engineering
Institute of Structural Analysis and Aseismic Research

myBiaxial
version 2.0

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Contents

1. Introduction

1.1 Scope

This document analyzes the features and usage of myBiaxial version 2.0. In addition, there is a brief description of the theory and a set of comprehensive examples.

1.2 Program requirements

The minimum requirements are:

- Operating System: Microsoft® Windows 95/98/ME/NT/2000/XP
- Visual Basic 6 Service Pack 5 runtime libraries.

1.3 Abbreviations

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1.4 About myBiaxial

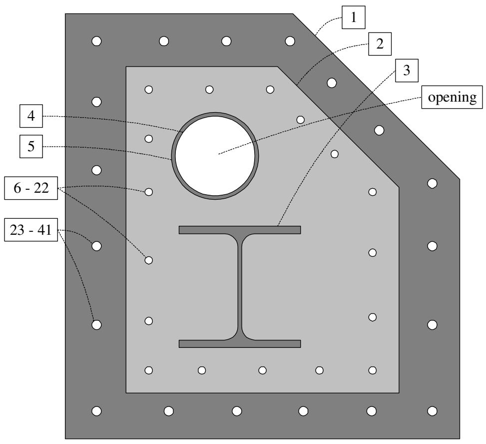
This program can analyze arbitrary cross-sections under combined biaxial bending and axial load. It can produce moment-curvature diagrams, interaction curves and failure surfaces. It can also calculate the deformed state of a cross-section under given external loads.

The program implements an algorithm that was presented in references [1] - [3].

2. Cross-section

2.1 General issues

The curvilinear polygon i.e. the polygon that has edges that are straight lines and/or arcs is the only type of graphical object used for the description of all cross-sections. For our purposes, each curvilinear polygon needs not be convex, but it must be simple i.e. two edges must not intersect in points other than its nodes and a single node cannot connect more than two edges. Also, the curvilinear polygons describing the cross-section must not intersect each other but one can fully include others. Since these polygons can be nested in any depth, it is obvious that any cross-section of interest can be described exactly. Circles are treated as two-sided curvilinear polygons with curved edges; the top and bottom quadrant points are used as nodes. Note that even small objects, such as the reinforcement bars, are treated as actual graphical objects and not dimensionless individual fibers.



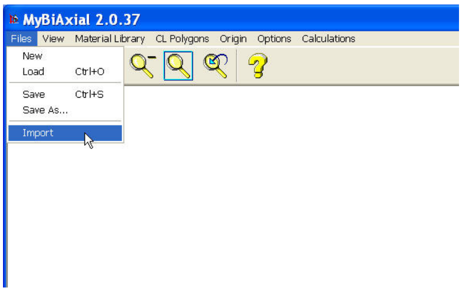
Object	Number of Nodes	Foreground material	Background material
1	5	Unconfined (outer) concrete	None
2	5	Confined (inner) concrete	Unconfined (outer) concrete
3	16	Structural steel	Confined (inner) concrete
4	2	None	Structural steel
5	2	Structural steel	Confined (inner) concrete
6 - 22	2	Reinforcement	Confined (inner) concrete
23 - 41	2	Reinforcement	Unconfined (outer) concrete

2.2 Data input

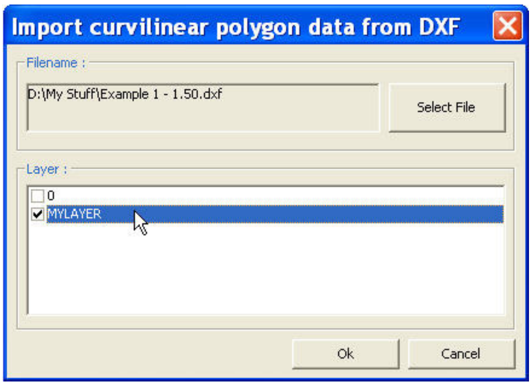
In order to create a cross-section, you need to draw the section using CAD software, such as AutoCad, Microstation, Intellicad etc. The following instructions apply to AutoCad version 2000 or later, but similar steps should be taken in case of other CAD packages.

- If you have already drawn the cross-section, open the existing drawing; if not, create a new drawing.
- Choose the units of the drawing (for example, mm); the choice of units in the drawing will affect the unit system of the results.
- Add a new separate layer which will contain all polylines and circles of the cross section. In the command line, type *layer [space]*. Press *New* to add a new layer and choose a unique name, i.e. *myLayer*. Make sure that the new layer is selected and press *Current* to make it the current layer; alternatively, double-click on the new layer.
- Draw the cross section using closed polylines and circles.
- Draw a new circle by typing *circle [space]* in the command line. Click on the drawing to select the center of the circle (alternatively, type the coordinates in the command line) and then type the radius of the circle in the command line.
- Draw a new polyline by typing *pline [space]*. Add straight lines or arcs using the available commands of the CAD software. When finished, close the polyline by selecting *close [space].*
- Note that a set of independent lines and/or arcs connected to form a closed polyline will *not* be recognized by myBiAxial. To fix this, use the polyline edit command by typing *pedit [space]* in the command line and then click on one of the lines. When prompted to convert one of the lines to a polyline press *[space]* to accept the default response ("yes"). Then type *join [space]* and select all other segments. When finished press *[space]*. The segments will be joined to form a single polyline.
- To make sure that a polyline is considered closed, select it in the drawing (make sure that it is the only object selected) and type *properties [space]* in the command line. This will produce the properties form (if it is not already visible). Make sure that the Boolean property *closed* (under "misc" category) is set to true.

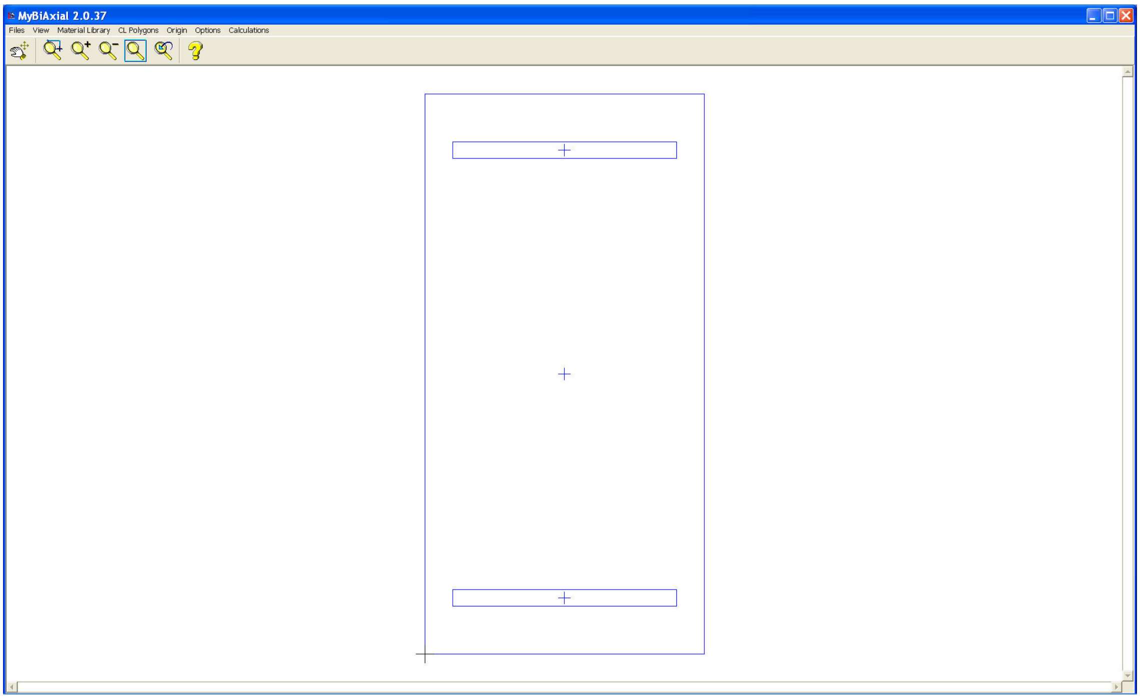
- For our purposes, each polyline needs not be convex, but it must be simple i.e. two edges must not intersect in points other than its nodes and a single node cannot connect more than two edges. Finally, all objects (polylines and circles) must not intersect each other but one can fully include others.
- Save the file as an *AutoCad 2000 DXF file*, *NOT* Autocad R12 DXF file.
- Open myBiaxial and select *Files > Import:*



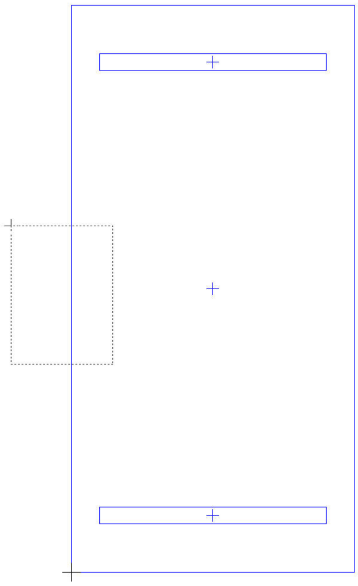
- Select the DXF file, select the layers containing the cross section (in this case, only *myLayer*) and press *Ok*:



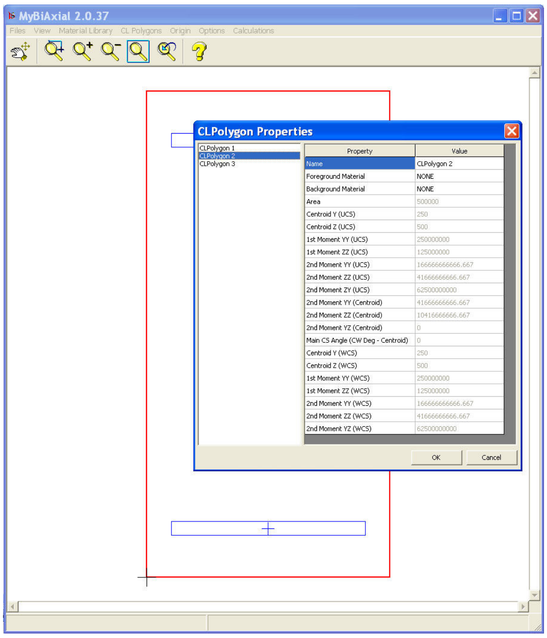
- The cross section will be loaded as shown in the next figure:



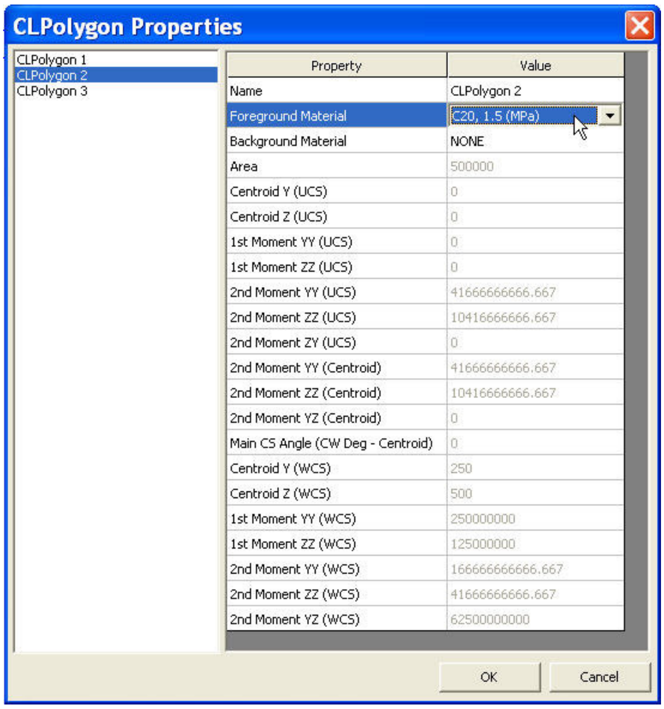
- Select the rectangle representing the concrete area by creating a right-to-left window that intersects it:



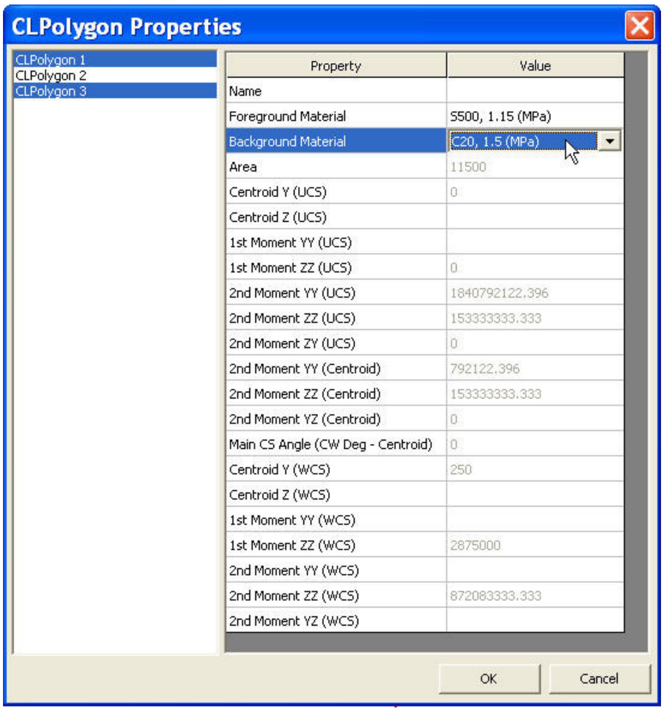
- Having this object selected, select *CL Polygons* from the menu. In this case, *CLPolygon2* is selected which means that the concrete rectangle corresponds to object *CLPolygon2.*



- Select *C20, 1.5 (MPa)* as the foreground material and *NONE* as the background material:



- From the list on the left, select *CLPolygon1,* and then add *CLPolygon3* (holding down the CTRL key). These are the remaining rectangles, i.e. the reinforcement. For these objects, select *S500, 1.15 (MPa)* as the foreground material and *C20, 1.5 (MPa)* as the background material:



- Press *Ok*. The cross-section is now fully defined and you can save the project by selecting *Files > Save As…*
- In this case, the drawing was defined in mm and the materials were defined in MPa. Therefore, the results will be given in MPa x mm^2 = 10^{-3} kN for the forces and MPa x mm^3 = 10^{-6} kNm for the bending moments. You can use the unit conversion factors (in this case, 10^{-3} for the forces and 10^{-6} for the bending moments for kN and kNm respectively) in the various calculations to work with familiar unit systems.

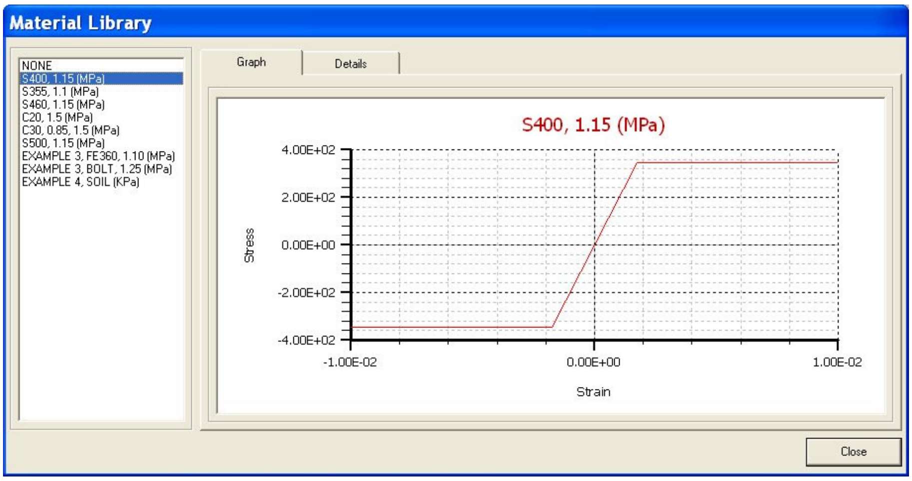
3. Material data

3.1 General issues

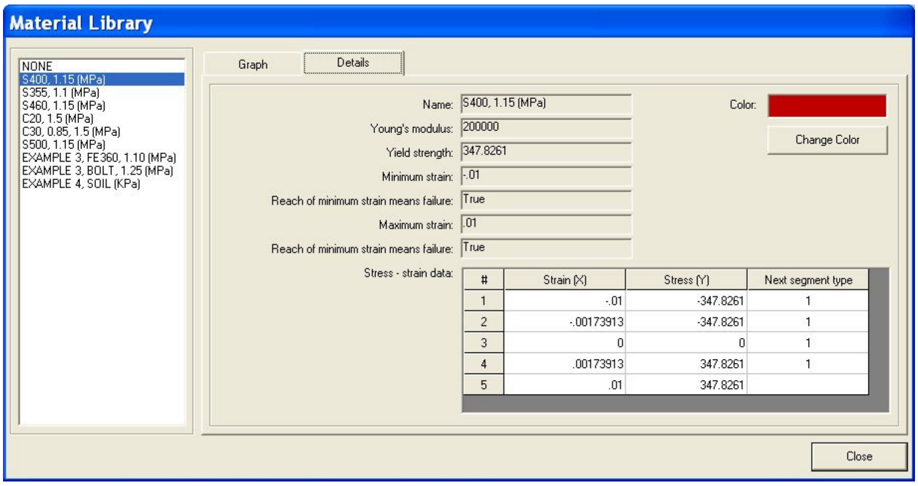
All material data is stored in a "material library". This library contains materials that can be used as "foreground" or "background" material for the curvilinear polygons defining the cross-section. It also contains a void material ("NONE") which is always used for the outermost curvilinear polygon as "background". It can also be used for inner curvilinear polygons in order to define holes (openings), as demonstrated in the examples.

3.2 Material library

The material library form can be found under the *"Material Library"* menu. The following form will appear:



On the left there is a list with all available materials. Click on a material to view the stress – strain diagram in the "*Graph*" tab on the right. Click on the "*Details*" tab to view the details of the material. You can also change the color of the material by clicking on the "*Change color*" command button. The changes will be saved in the file when the program exits.



In order to add/delete/modify a material, you need to modify the file "*material.lib*" which can be found at the installation folder. This file has a certain format, described later in this chapter.

3.3 "Material.lib": File Format

This file is a standard ASCII text file. You must use a dot "." as the decimal symbol.

Each material is defined as follows:

- Line 1: Title (string).
- Line 2: RGB Color code (long integer). It can be set to "0" for black, and it can be modified later within the program.
- Line 3: Young's modulus. It is used for the calculation of the elastic centroid of the section.
- Line 4: Yield strength. It is used for the calculation of the plastic centroid of the section.
- Line 5: One of "true", "false". Boolean indicating whether the reach of maximum tensile strain signifies the failure of the cross section. Tension is positive, compression is negative.
- Line 6: maximum tensile strain.

- Line 7: One of "true", "false". Boolean indicating whether the reach of maximum compressive strain signifies the failure of the cross section. Tension is positive, compression is negative.
- Line 8: maximum compressive strain.
- Line 9: Number of points in the stress – strain diagram. Must be more than or equal to 2.
- Next lines: The stress – strain data with segment type codes between them. The sequence is as follows:
  - Strain
  - Stress
  - Next segment code (omit this entry for the last point). It can be one of the following:
    - "1": The next segment is linear.
    - "21": The next segment is the first segment of a parabolic sequence.
    - "22": The next segment is the second segment of a parabolic sequence.
    - "31": The next segment is the first segment of a cubic sequence.
    - "32": The next segment is the second segment of a cubic sequence.
    - "33": The next segment is the third segment of a cubic sequence.

For example, a certain material might look like this:

```
S400, 1.15 (MPa)
192
200000
347.8261
True
.01
True
-.01
```

```
5
-.01
-347.8261
1
-.00173913
-347.8261
1
0
0
1
.00173913
347.8261
1
.01
347.8261
```

The file ("material.lib") contains a set of materials, such as the example above, with no whitespaces or line feeds in between. Make sure that each material has a unique name.

4. Comprehensive Examples

4.1 Eurocode 2 design charts

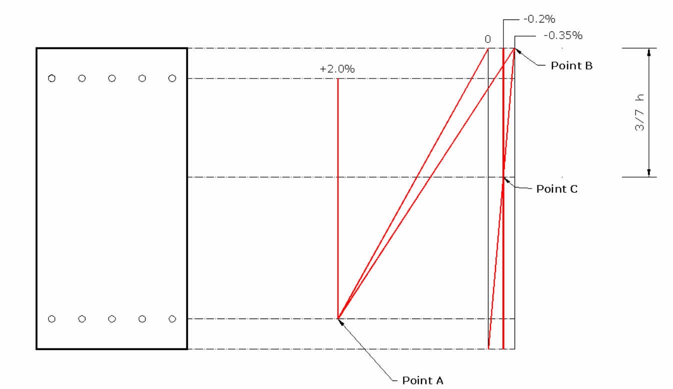
Eurocode 2 provides design charts for common reinforced concrete cross sections. These charts provide combinations of axial loads and their respective ultimate bending moment capacities (which correspond to the conventional failure of the cross section), for a range of longitudinal reinforcement expressed by the mechanical reinforcement percentage ω (equation 1).

$$\omega = \frac{A_{s,tot}}{A_{c,tot}} \cdot \frac{f_{yd}}{f_{cd}} \quad (1)$$

where $A_{s,tot}$ is the total area of longitudinal reinforcement, $A_{c,tot}$ is the total area of concrete, f_{yd}, f_{cd} are the design strengths of steel and concrete respectively. Also, the axial load and bending moment are normalized with respect to the concrete properties and the cross sectional dimensions (equation 2); therefore, a single chart covers all cases for a certain steel grade.

$$\nu = \frac{N_d}{A_{c,tot} \cdot f_{cd}} \qquad \mu = \frac{M_d}{A_{c,tot} \cdot h \cdot f_{cd}} \quad (2)$$

Eurocode 2 specifies the value of 0.020 as the ultimate strain limit for longitudinal steel reinforcement. Also, for large compressive axial loads, it reduces the ultimate curvature capacity by imposing the rotation of the strain profile around point C which is located at a distance 3/7·h from the most compressed fiber and has a strain of ε_0=-0.002.



This restriction is included easily in the algorithm; however, it is of little practical interest since large compressive axial loads in concrete cross sections must be avoided for other reasons i.e. creep.

The developed computer program was used to calculate pairs of axial loads and bending moments for the rectangular cross section of Figure 1a. The characteristic strengths and partial safety factors for concrete and reinforcement bars were taken as follows:

$$f_{ck} = 20MPa, \gamma_c = 1.5$$
$$f_y = 500MPa, \gamma_r = 1.15$$

Five different cases of longitudinal reinforcement were considered, i.e. ω=0.00, 0.50, 1.00, 1.50, 2.00. The computed results, summarized in Table 1, follow the corresponding curve exactly, as shown in Figure 1c.

v	μ (ω=0.00)	μ (ω=0.50)	μ (ω=1.00)	μ (ω=1.50)	μ (ω=2.00)
1.60					0.1607
1.40				0.0402	0.2408
1.20				0.1203	0.3219
1.00				0.2007	0.4031
0.80			0.0801	0.2823	0.4841
0.60			0.1613	0.3636	0.5645
0.40		0.0400	0.2433	0.4440	0.6441
0.20		0.1228	0.3237	0.5232	0.7230
0.00	0.0000	0.2031	0.4020	0.6015	0.8016
-0.10	0.0424	0.2412	0.4406	0.6402	0.8397
-0.20	0.0746	0.2748	0.4739	0.6728	0.8717
-0.30	0.0951	0.2939	0.4920	0.6903	0.8883
-0.35	0.1010	0.2988	0.4967	0.6944	0.8919
-0.40	0.1033	0.2943	0.4883	0.6828	0.8775
-0.60	0.0824	0.2465	0.4287	0.6176	0.8091
-0.80	0.0193	0.1938	0.3690	0.5526	0.7409
-1.00		0.1292	0.3072	0.4875	0.6729
-1.20		0.0548	0.2406	0.4214	0.6047
-1.40			0.1670	0.3525	0.5358
-1.60			0.0897	0.2792	0.4652
-1.80				0.2030	0.3921
-2.00				0.1245	0.3159
-2.20					0.2384
-2.40					0.1595

Table 1. Computed results for example 1 (summary)

More specifically, the results for each case are as follows:

v	F (N)	M (Nmm)	μ	φ (mm^{-1})	ε_0
0.00	0.000E+00	0.000E+00	0.0000	0.000E+00	0.000E+00
-0.10	-6.667E+05	2.827E+08	0.0424	7.000E-06	2.022E-03
-0.20	-1.333E+06	4.970E+08	0.0746	7.000E-06	1.186E-03
-0.30	-2.000E+06	6.342E+08	0.0951	7.000E-06	3.627E-04
-0.35	-2.333E+06	6.730E+08	0.1010	6.875E-06	-6.005E-05
-0.40	-2.667E+06	6.884E+08	0.1033	6.000E-06	-4.902E-04
-0.60	-4.000E+06	5.492E+08	0.0824	4.014E-06	-1.493E-03
-0.80	-5.333E+06	1.288E+08	0.0193	2.747E-06	-2.124E-03

Table 2. Example 1 results (ω=0.00)

v	F (N)	M (Nmm)	μ	φ (mm^{-1})	ε_0
0.40	2.667E+06	2.667E+08	0.0400	1.080E-05	5.626E-03
0.20	1.333E+06	8.189E+08	0.1228	1.248E-05	4.946E-03
0.00	0.000E+00	1.354E+09	0.2031	1.392E-05	4.367E-03
-0.10	-6.667E+05	1.608E+09	0.2412	1.450E-05	4.011E-03
-0.20	-1.333E+06	1.832E+09	0.2748	1.163E-05	2.321E-03
-0.30	-2.000E+06	1.959E+09	0.2939	7.852E-06	4.278E-04
-0.35	-2.333E+06	1.992E+09	0.2988	6.755E-06	-1.224E-04
-0.40	-2.667E+06	1.962E+09	0.2943	6.156E-06	-4.218E-04
-0.60	-4.000E+06	1.643E+09	0.2465	5.053E-06	-9.743E-04
-0.80	-5.333E+06	1.292E+09	0.1938	4.137E-06	-1.432E-03
-1.00	-6.667E+06	8.611E+08	0.1292	3.394E-06	-1.820E-03
-1.20	-8.000E+06	3.650E+08	0.0548	2.519E-06	-2.241E-03

Table 3. Example 1 results (ω=0.50)

v	F (N)	M (Nmm)	μ	φ (mm^{-1})	ε_0
0.80	5.333E+06	5.338E+08	0.0801	1.074E-05	5.601E-03
0.60	4.000E+06	1.075E+09	0.1613	1.174E-05	5.193E-03
0.40	2.667E+06	1.622E+09	0.2433	1.261E-05	4.836E-03
0.20	1.333E+06	2.158E+09	0.3237	1.347E-05	4.481E-03
0.00	0.000E+00	2.680E+09	0.4020	1.400E-05	3.990E-03
-0.10	-6.667E+05	2.937E+09	0.4406	1.475E-05	3.931E-03
-0.20	-1.333E+06	3.159E+09	0.4739	1.128E-05	2.147E-03
-0.30	-2.000E+06	3.280E+09	0.4920	7.691E-06	3.465E-04
-0.35	-2.333E+06	3.311E+09	0.4967	6.634E-06	-1.830E-04
-0.40	-2.667E+06	3.255E+09	0.4883	6.188E-06	-4.052E-04
-0.60	-4.000E+06	2.858E+09	0.4287	5.469E-06	-7.648E-04
-0.80	-5.333E+06	2.460E+09	0.3690	4.797E-06	-1.099E-03
-1.00	-6.667E+06	2.048E+09	0.3072	4.188E-06	-1.406E-03
-1.20	-8.000E+06	1.604E+09	0.2406	3.633E-06	-1.683E-03
-1.40	-9.333E+06	1.113E+09	0.1670	3.094E-06	-1.950E-03
-1.60	-1.067E+07	5.978E+08	0.0897	2.438E-06	-2.275E-03

Table 4. Example 1 results (ω=1.00)

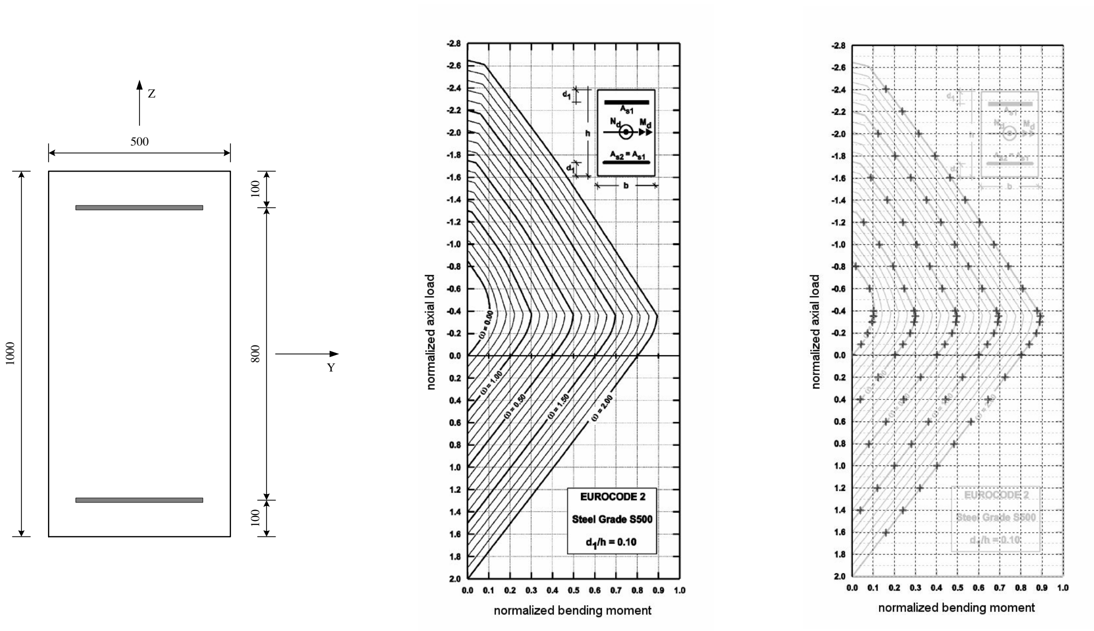
v	F (N)	M (Nmm)	μ	φ (mm^{-1})	ε_0
1.40	9.333E+06	2.682E+08	0.0402	9.966E-06	5.870E-03
1.20	8.000E+06	8.017E+08	0.1203	1.068E-05	5.575E-03
1.00	6.667E+06	1.338E+09	0.2007	1.137E-05	5.289E-03
0.80	5.333E+06	1.882E+09	0.2823	1.200E-05	5.027E-03
0.60	4.000E+06	2.424E+09	0.3636	1.262E-05	7.771E-03
0.40	2.667E+06	2.960E+09	0.4440	1.324E-05	4.514E-03
0.20	1.333E+06	3.488E+09	0.5232	1.386E-05	4.257E-03
0.00	0.000E+00	4.010E+09	0.6015	1.448E-05	4.000E-03
-0.10	-6.667E+05	4.268E+09	0.6402	1.444E-05	3.729E-03
-0.20	-1.333E+06	4.485E+09	0.6728	1.095E-05	1.981E-03
-0.30	-2.000E+06	4.602E+09	0.6903	7.535E-06	2.681E-04
-0.35	-2.333E+06	4.629E+09	0.6944	6.517E-06	-2.415E-04
-0.40	-2.667E+06	4.552E+09	0.6828	6.203E-06	-3.982E-04
-0.60	-4.000E+06	4.117E+09	0.6176	5.572E-06	-6.611E-04
-0.80	-5.333E+06	3.684E+09	0.5526	5.166E-06	-9.168E-04
-1.00	-6.667E+06	3.250E+09	0.4875	4.678E-06	-1.160E-03
-1.20	-8.000E+06	2.809E+09	0.4214	4.215E-06	-1.391E-03
-1.40	-9.333E+06	2.350E+09	0.3525	3.777E-06	-1.610E-03
-1.60	-1.067E+07	1.861E+09	0.2792	3.359E-06	-1.818E-03
-1.80	-1.200E+07	1.353E+09	0.2030	2.914E-06	-2.041E-03
-2.00	-1.333E+07	8.302E+08	0.1245	2.406E-06	-2.293E-03

Table 5. Example 1 results (ω=1.50)

v	F (N)	M (Nmm)	μ	φ (mm^{-1})	ε_0
1.60	1.067E+07	1.071E+09	0.1607	1.062E-05	5.550E-03
1.40	9.333E+06	1.605E+09	0.2408	1.114E-05	5.330E-03
1.20	8.000E+06	2.146E+09	0.3219	1.164E-05	5.123E-03
1.00	6.667E+06	2.687E+09	0.4031	1.211E-05	4.923E-03
0.80	5.333E+06	3.227E+09	0.4841	1.259E-05	4.721E-03
0.60	4.000E+06	3.763E+09	0.5645	1.308E-05	4.519E-03
0.40	2.667E+06	4.294E+09	0.6441	1.356E-05	4.318E-03
0.20	1.333E+06	4.820E+09	0.7230	1.400E-05	4.103E-03
0.00	0.000E+00	5.344E+09	0.8016	1.450E-05	3.911E-03
-0.10	-6.667E+05	5.598E+09	0.8397	1.384E-05	3.429E-03
-0.20	-1.333E+06	5.811E+09	0.8717	1.064E-05	1.824E-03
-0.30	-2.000E+06	5.922E+09	0.8883	7.358E-06	1.929E-04
-0.35	-2.333E+06	5.946E+09	0.8919	6.414E-06	-2.932E-04
-0.40	-2.667E+06	5.850E+09	0.8775	6.215E-06	-3.926E-04
-0.60	-4.000E+06	5.394E+09	0.8091	5.795E-06	-6.018E-04
-0.80	-5.333E+06	4.939E+09	0.7409	5.387E-06	-8.052E-04
-1.00	-6.667E+06	4.486E+09	0.6729	4.991E-06	-1.004E-03
-1.20	-8.000E+06	4.031E+09	0.6047	4.605E-06	-1.197E-03
-1.40	-9.333E+06	3.572E+09	0.5358	4.233E-06	-1.383E-03
-1.60	-1.067E+07	3.101E+09	0.4652	3.875E-06	-1.562E-03
-1.80	-1.200E+07	2.614E+09	0.3921	3.523E-06	-1.737E-03
-2.00	-1.333E+07	2.106E+09	0.3159	3.176E-06	-1.912E-03

-2.20	-1.467E+07	1.589E+09	0.2384	2.797E-06	-2.099E-03
-2.40	-1.600E+07	1.063E+09	0.1595	2.388E-06	-2.303E-03

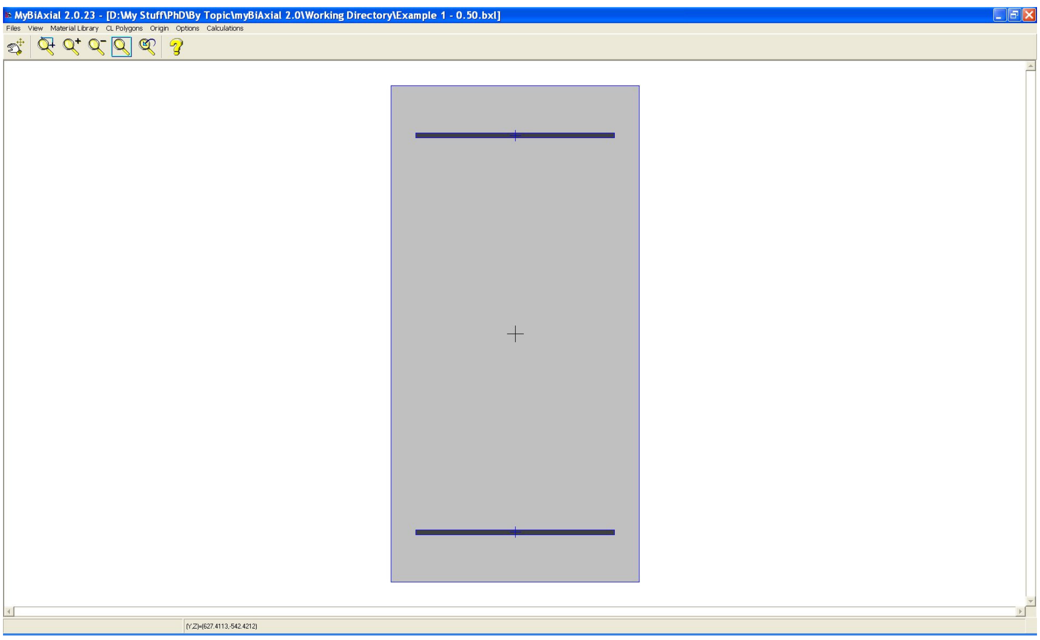
Table 6. Example 1 results (ω=2.00)



(a) (b) (c)

Figure 1. (a) Rectangular reinforced concrete cross section (distances in mm)
(b) Corresponding EC2 design chart (steel grade S500)
(c) Results from proposed algorithm superimposed over the design chart

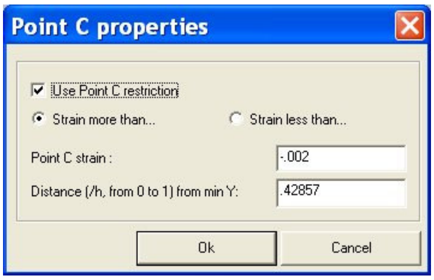
The program setup includes this example (for ω = 0.50) and the material data necessary to replicate the results. Select *Files > Load* and select the file *Example 1 - 0.50.bxl* which can be found in the application path. The main form will look like this:



The cross-section was drawn in mm whereas the material stresses were defined in MPa. Therefore the results for force will be in MPa x mm^2 = N whereas the results for moment will be in MPa x mm^3= Nmm.

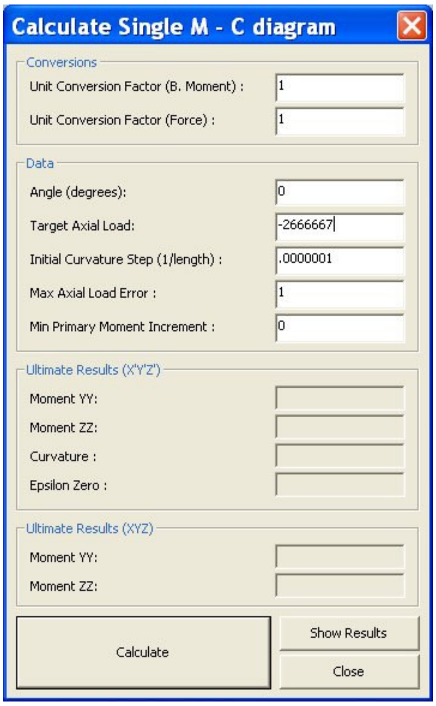
In order to obtain the results in other units, you should define conversion factors for bending moments and forces. In this case, a conversion factor of 1E-06 and 1E-03 for moments and forces will yield results in kNm and kN respectively.

Select *Calculations > Point C Restrictions* and input the data as follows (3/7 = 0.42857):

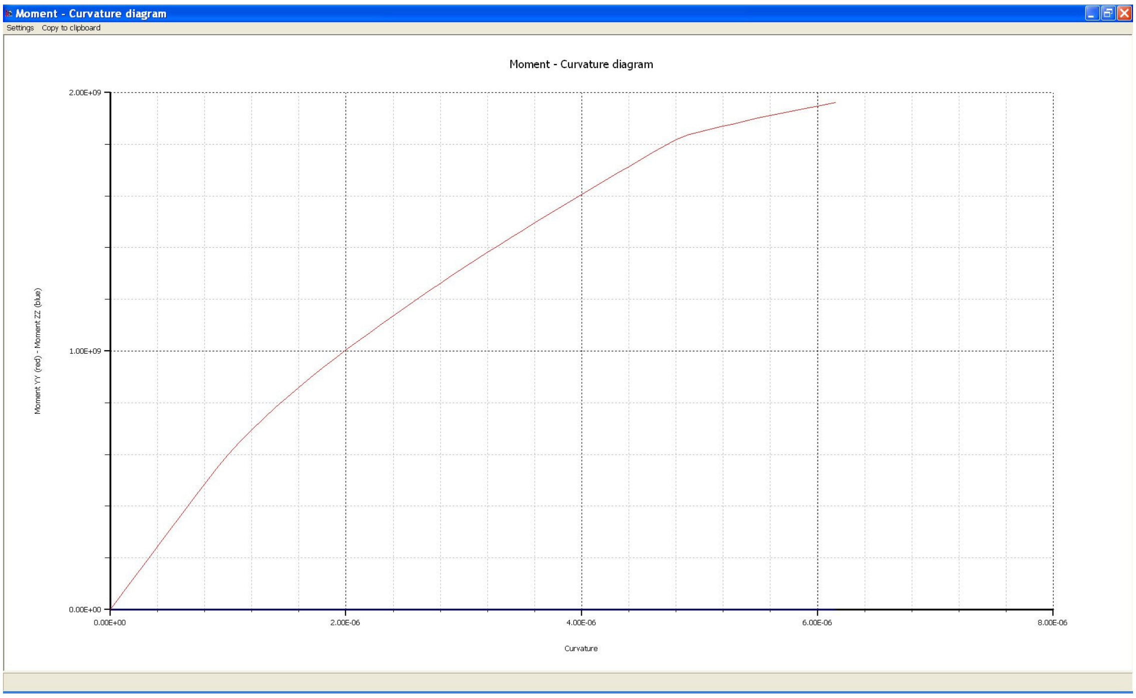


Select *Calculations > Single Moment – Curvature Diagram*. As an example, we will verify the case for v = -0.40, which means that the axial load will be equal to -2.667E+06 (compressive) at all times.

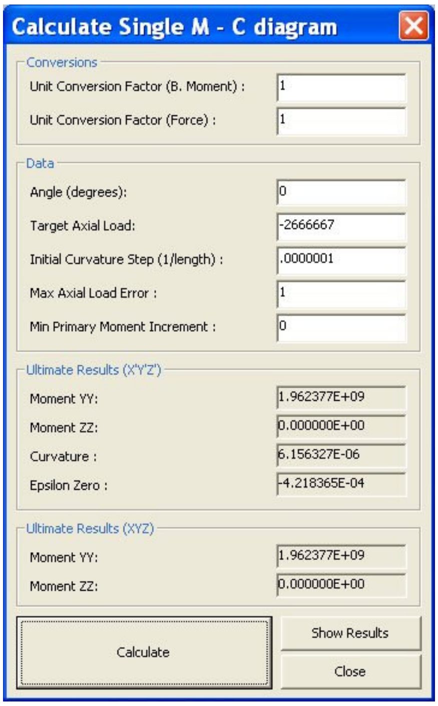
Input the data as shown in the following figure:



Press *Calculate* to calculate the moment – curvature diagram. The diagram will look like this:



The ultimate results are shown in the form:



Click on *Show Results* to open the calculation log. This provides information for the intermediate steps as well as additional information for the final step. For this case, the calculation log is shown in the following table. Note the highlighted strain value of -3.500E-3 at the final step, which means that the concrete has failed in compression.

```
******************************************************************************
Moment - Curvature Diagram
******************************************************************************

ATTENTION: Bending Moment unit converter is 1
ATTENTION: Axial Force unit converter is 1

Moment - Curvature diagram for axial load = -2666667
Angle (degrees) = 0,000
```

Counter	EpsilonZero	Curvature	Fx	MomentY	MomentZ	Iterations
1	-4,074520564E-04	0,000000000E+00	-2,666667477E+06	1,809699825E-06	0,000000000E+00	7
2	-4,076437777E-04	1,000000000E-07	-2,666667423E+06	6,102489540E+07	0,000000000E+00	7
3	-4,082190628E-04	2,000000000E-07	-2,666667370E+06	1,220234243E+08	0,000000000E+00	5
4	-4,091781678E-04	3,000000000E-07	-2,666667000E+06	1,829692000E+08	0,000000000E+00	6
5	-4,105217240E-04	4,000000000E-07	-2,666667000E+06	2,438357777E+08	0,000000000E+00	6
6	-4,122503944E-04	5,000000000E-07	-2,666667000E+06	3,045966512E+08	0,000000000E+00	6
7	-4,143651143E-04	6,000000000E-07	-2,666667000E+06	3,652252160E+08	0,000000000E+00	6
8	-4,168670321E-04	7,000000000E-07	-2,666667000E+06	4,256947480E+08	0,000000000E+00	6
9	-4,197575124E-04	8,000000000E-07	-2,666667000E+06	4,859783778E+08	0,000000000E+00	6
10	-4,226391262E-04	9,000000000E-07	-2,666666985E+06	5,449715709E+08	0,000000000E+00	6
11	-4,239075317E-04	1,000000000E-06	-2,666666997E+06	5,986353973E+08	0,000000000E+00	6
12	-4,238251673E-04	1,100000000E-06	-2,666666485E+06	6,481358224E+08	0,000000000E+00	5
13	-4,227588455E-04	1,200000000E-06	-2,666666056E+06	6,947023261E+08	0,000000000E+00	5
14	-4,207319702E-04	1,300000000E-06	-2,666667000E+06	7,386507829E+08	0,000000000E+00	6
15	-4,179110487E-04	1,400000000E-06	-2,666667000E+06	7,805024940E+08	0,000000000E+00	6
16	-4,144315189E-04	1,500000000E-06	-2,666667000E+06	8,206434701E+08	0,000000000E+00	6
17	-4,104046070E-04	1,600000000E-06	-2,666667000E+06	8,593650943E+08	0,000000000E+00	6
18	-4,059230104E-04	1,700000000E-06	-2,666667000E+06	8,968910079E+08	0,000000000E+00	6
19	-4,010649174E-04	1,800000000E-06	-2,666667000E+06	9,333953276E+08	0,000000000E+00	6
20	-3,958970050E-04	1,900000000E-06	-2,666667000E+06	9,690152920E+08	0,000000000E+00	6
21	-3,904767110E-04	2,000000000E-06	-2,666667000E+06	1,003860227E+09	0,000000000E+00	6
22	-3,848539838E-04	2,100000000E-06	-2,666667000E+06	1,038018016E+09	0,000000000E+00	6
23	-3,790726481E-04	2,200000000E-06	-2,666667000E+06	1,071559858E+09	0,000000000E+00	6
24	-3,731714839E-04	2,300000000E-06	-2,666667000E+06	1,104543797E+09	0,000000000E+00	6
25	-3,671850883E-04	2,400000000E-06	-2,666667000E+06	1,137017405E+09	0,000000000E+00	6
26	-3,611445723E-04	2,500000000E-06	-2,666667000E+06	1,169019812E+09	0,000000000E+00	6
27	-3,550781286E-04	2,600000000E-06	-2,666667000E+06	1,200583290E+09	0,000000000E+00	6
28	-3,490114996E-04	2,700000000E-06	-2,666667000E+06	1,231734477E+09	0,000000000E+00	6
29	-3,429683653E-04	2,800000000E-06	-2,666667000E+06	1,262495348E+09	0,000000000E+00	6
30	-3,369706692E-04	2,900000000E-06	-2,666667000E+06	1,292883974E+09	0,000000000E+00	6
31	-3,310388936E-04	3,000000000E-06	-2,666667000E+06	1,322915140E+09	0,000000000E+00	6
32	-3,251920813E-04	3,100000000E-06	-2,666666289E+06	1,352600793E+09	0,000000000E+00	5
33	-3,194489765E-04	3,200000000E-06	-2,666666590E+06	1,381950619E+09	0,000000000E+00	5
34	-3,138266372E-04	3,300000000E-06	-2,666666811E+06	1,410972060E+09	0,000000000E+00	5
35	-3,083417166E-04	3,400000000E-06	-2,666666932E+06	1,439670839E+09	0,000000000E+00	5
36	-3,030040221E-04	3,500000000E-06	-2,666666945E+06	1,468059845E+09	0,000000000E+00	5
37	-2,978087444E-04	3,600000000E-06	-2,666666956E+06	1,496169403E+09	0,000000000E+00	5
38	-2,927493197E-04	3,700000000E-06	-2,666666959E+06	1,524028572E+09	0,000000000E+00	5
39	-2,878195307E-04	3,800000000E-06	-2,666666956E+06	1,551662779E+09	0,000000000E+00	5
40	-2,830134838E-04	3,900000000E-06	-2,666666952E+06	1,579094359E+09	0,000000000E+00	5
41	-2,783255855E-04	4,000000000E-06	-2,666666948E+06	1,606343002E+09	0,000000000E+00	5

Table 7. Example 1: Calculation log (1 of 2)

42	-2,737505249E-04	4,100000000E-06	-2,666666943E+06	1,633426129E+09	0,000000000E+00	5
43	-2,692832565E-04	4,200000000E-06	-2,666666937E+06	1,660359206E+09	0,000000000E+00	5
44	-2,649189935E-04	4,300000000E-06	-2,666666945E+06	1,687155996E+09	0,000000000E+00	5
45	-2,606553825E-04	4,400000000E-06	-2,666667000E+06	1,713826568E+09	0,000000000E+00	5
46	-2,564944324E-04	4,500000000E-06	-2,666667000E+06	1,740375580E+09	0,000000000E+00	5
47	-2,524337987E-04	4,600000000E-06	-2,666667000E+06	1,766811357E+09	0,000000000E+00	5
48	-2,484698969E-04	4,700000000E-06	-2,666667000E+06	1,793142658E+09	0,000000000E+00	5
49	-2,452285918E-04	4,800000000E-06	-2,666667006E+06	1,818753056E+09	0,000000000E+00	6
50	-2,515449172E-04	4,900000000E-06	-2,666667000E+06	1,835017940E+09	0,000000000E+00	4
51	-2,626318793E-04	5,000000000E-06	-2,666667000E+06	1,846758221E+09	0,000000000E+00	4
52	-2,741817722E-04	5,100000000E-06	-2,666667000E+06	1,858182040E+09	0,000000000E+00	4
53	-2,861835660E-04	5,200000000E-06	-2,666667000E+06	1,869302102E+09	0,000000000E+00	4
54	-2,986265787E-04	5,300000000E-06	-2,666667000E+06	1,880130367E+09	0,000000000E+00	4
55	-3,115004621E-04	5,400000000E-06	-2,666667000E+06	1,890678108E+09	0,000000000E+00	4
56	-3,247951892E-04	5,500000000E-06	-2,666667000E+06	1,900955966E+09	0,000000000E+00	4
57	-3,385010418E-04	5,600000000E-06	-2,666667000E+06	1,910974002E+09	0,000000000E+00	4
58	-3,526085984E-04	5,700000000E-06	-2,666667000E+06	1,920741741E+09	0,000000000E+00	4
59	-3,671087235E-04	5,800000000E-06	-2,666667000E+06	1,930268210E+09	0,000000000E+00	4
60	-3,819925563E-04	5,900000000E-06	-2,666667000E+06	1,939561976E+09	0,000000000E+00	4
61	-3,972515008E-04	6,000000000E-06	-2,666667000E+06	1,948631178E+09	0,000000000E+00	4
62	-4,128772157E-04	6,100000000E-06	-2,666667000E+06	1,957483557E+09	0,000000000E+00	4
63	-4,208250729E-04	6,150000000E-06	-2,666667000E+06	1,961830754E+09	0,000000000E+00	4
64	-4,218248101E-04	6,156250000E-06	-2,666667000E+06	1,962370516E+09	0,000000000E+00	4
65	-4,218326260E-04	6,156298828E-06	-2,666667000E+06	1,962374730E+09	0,000000000E+00	4
66	-4,218365340E-04	6,156323242E-06	-2,666667000E+06	1,962376837E+09	0,000000000E+00	4
67	-4,218368530E-04	6,156326294E-06	-2,666666714E+06	1,962377146E+09	0,000000000E+00	3
68	-4,218365340E-04	6,156326675E-06	-2,666666073E+06	1,962377281E+09	0,000000000E+00	1
69	-4,218365669E-04	6,156326866E-06	-2,666666077E+06	1,962377297E+09	0,000000000E+00	3
70	-4,218365340E-04	6,156326914E-06	-2,666666008E+06	1,962377312E+09	0,000000000E+00	1
71	-4,218365371E-04	6,156326926E-06	-2,666666010E+06	1,962377313E+09	0,000000000E+00	3
72	-4,218365340E-04	6,156326932E-06	-2,666666004E+06	1,962377315E+09	0,000000000E+00	1
73	-4,218365327E-04	6,156326935E-06	-2,666666001E+06	1,962377315E+09	0,000000000E+00	3
74	-4,218365325E-04	6,156326935E-06	-2,666666000E+06	1,962377316E+09	0,000000000E+00	3
75	-4,218365324E-04	6,156326935E-06	-2,666666000E+06	1,962377316E+09	0,000000000E+00	3
76	-4,218365324E-04	6,156326935E-06	-2,666666000E+06	1,962377316E+09	0,000000000E+00	3
77	-4,218365324E-04	6,156326935E-06	-2,666666000E+06	1,962377316E+09	0,000000000E+00	3
78	-4,218365324E-04	6,156326935E-06	-2,666666000E+06	1,962377316E+09	0,000000000E+00	3
79	-4,218365324E-04	6,156326935E-06	-2,666666000E+06	1,962377316E+09	0,000000000E+00	3
80	-4,218365324E-04	6,156326935E-06	-2,666666000E+06	1,962377316E+09	0,000000000E+00	3
81	-4,218365324E-04	6,156326935E-06	-2,666666000E+06	1,962377316E+09	0,000000000E+00	3

Strains at final step:

# CLTrapezoid	Foreground Material	Background Material	Z bottom	Z top	Min Strain	Max Strain
1	S500, 1.15 (MPa)	C20, 1.5 (MPa)	395,208333000	404,791667000	2,011195173E-03	2,070193310E-03
2	S500, 1.15 (MPa)	C20, 1.5 (MPa)	-404,791667000	-395,208333000	-2,913866375E-03	-2,854868238E-03
3	C20, 1.5 (MPa)	NONE	-500,000000000	500,000000000	-3,500000000E-03	2,656326935E-03

Table 8. Example 1: Calculation log (2 of 2)

4.2 Arbitrary cross-section

This is an example presented by Chen et al. [5], which invokes the polygonal composite column cross section of Figure 2. The cross section consists of a concrete core, an asymmetrically placed H – shaped steel section, 15 reinforcement bars of diameter 18mm and a circular opening.

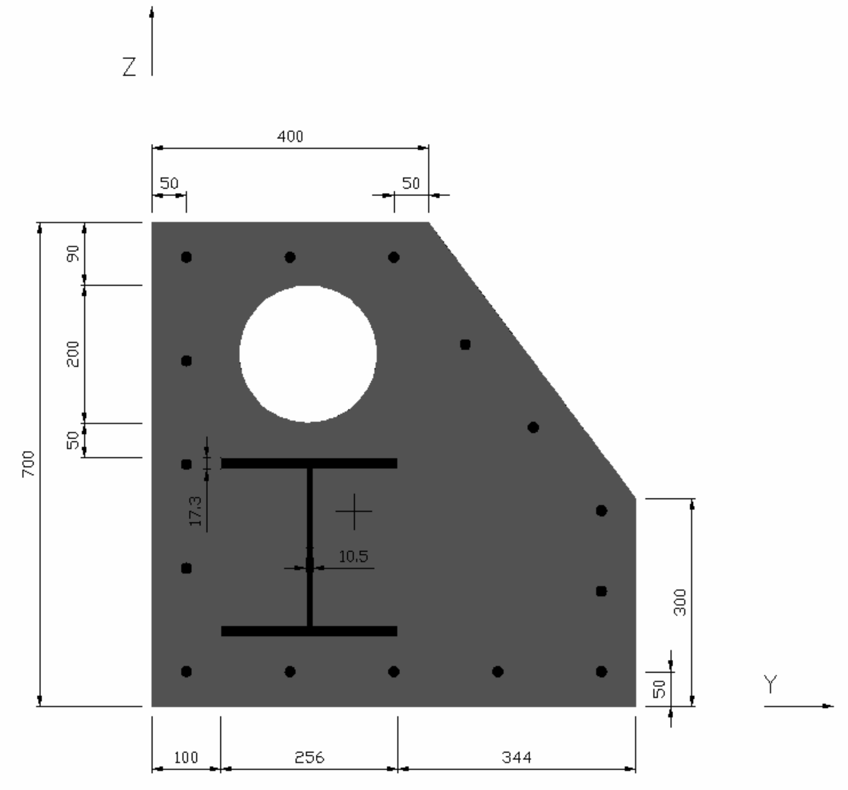


Figure 2. Composite column cross section

Chen et al. use a quasi – Newton method [13] to analyze the cross section. However, the convergence of the iterative process invoked by this algorithm cannot be guaranteed when dealing with large axial loads i.e. loads that approach the axial load capacity under pure compression. In order to ensure the stability of Chen's algorithm, the plastic centroid must be used as the origin of the Cartesian system. For an arbitrary cross section, the plastic centroid can be calculated as follows:

$$Y_{pc} = \frac{Y_c \cdot A_c \cdot f_{cc}/\gamma_c + Y_s \cdot A_s \cdot f_s/\gamma_s + Y_r \cdot A_r \cdot f_r/\gamma_r}{A_c \cdot f_{cc}/\gamma_c + A_s \cdot f_s/\gamma_s + A_r \cdot f_r/\gamma_r}$$

$$Z_{pc} = \frac{Z_c \cdot A_c \cdot f_{cc}/\gamma_c + Z_s \cdot A_s \cdot f_s/\gamma_s + Z_r \cdot A_r \cdot f_r/\gamma_r}{A_c \cdot f_{cc}/\gamma_c + A_s \cdot f_s/\gamma_s + A_r \cdot f_r/\gamma_r}$$

where, A_c, A_r, A_s are the total areas of concrete, reinforcing bars and structural steel respectively; f_{cc}, f_r, f_s, are the respective characteristic strengths; γ_{cc}, γ_r, γ_s are the respective partial safety factors, Y_c, Z_c, Y_r, Z_r, Y_s, Z_s, are the coordinates of the respective centroids. In this case, the coordinates of the plastic centroid with respect to the bottom left corner are [5] Y_{pc}=292.2mm, Z_{pc}=281.5mm.

The stress – strain curve for concrete (CEC 1994) which consists of a parabolic and a linear (horizontal) part was used in the calculation, with $f_{cc}=0.85 \cdot f_{ck}/\gamma_c$, $\varepsilon_0=0.002$ and $\varepsilon_{cu}=0.0035$. The Young modulus for all steel sections was 200GPa while the maximum strain was $\varepsilon_u=\pm0.010$.

The characteristic strengths and partial safety factors for concrete, structural steel and reinforcement bars were taken as follows:

$$f_{ck} = 30MPa, \gamma_c = 1.5$$
$$f_s = 355MPa, \gamma_s = 1.1$$
$$f_y = 460MPa, \gamma_r = 1.15$$

The analysis was carried out with an angle step of 5 degrees and an initial curvature step of 1E-06.

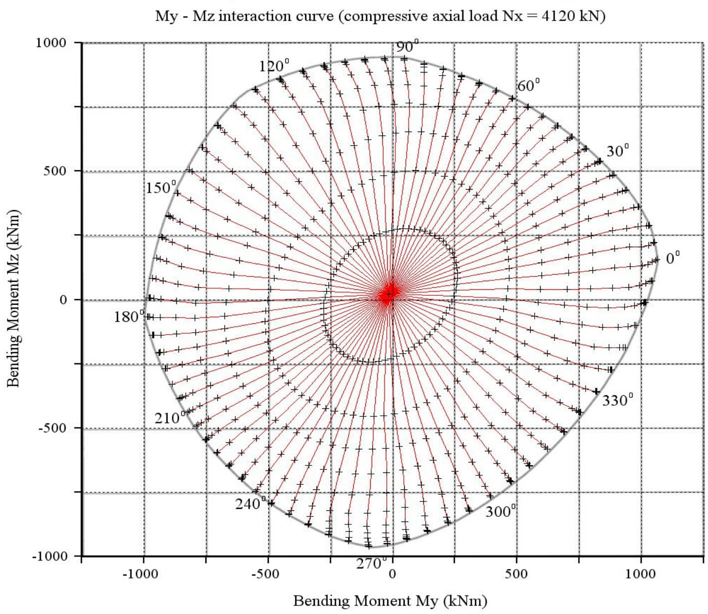


Figure 3. Interaction curve for compressive axial load 4120 kN

Figure 3 shows the interaction curve produced by the proposed algorithm for compressive axial load 4120kN. The image is superimposed over the results taken from [5]; it is obvious that the curves almost coincide. The same figure also shows the paths of the analyses and the directions of the neutral axes that correspond to each spike. Note that the data for each spike becomes denser near failure; this is because the curvature step is decreased in order to achieve accuracy. By repeating this procedure for various axial loads we obtain the complete failure surface of Figure 4.

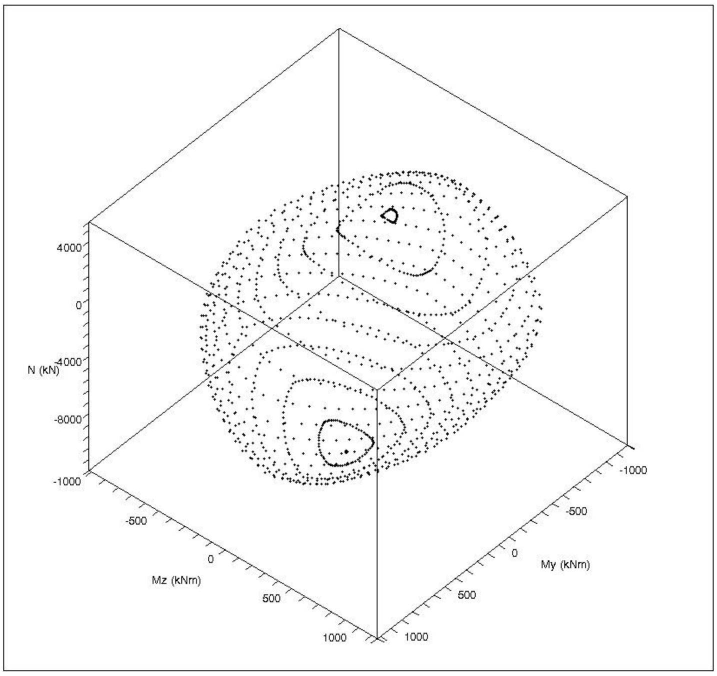
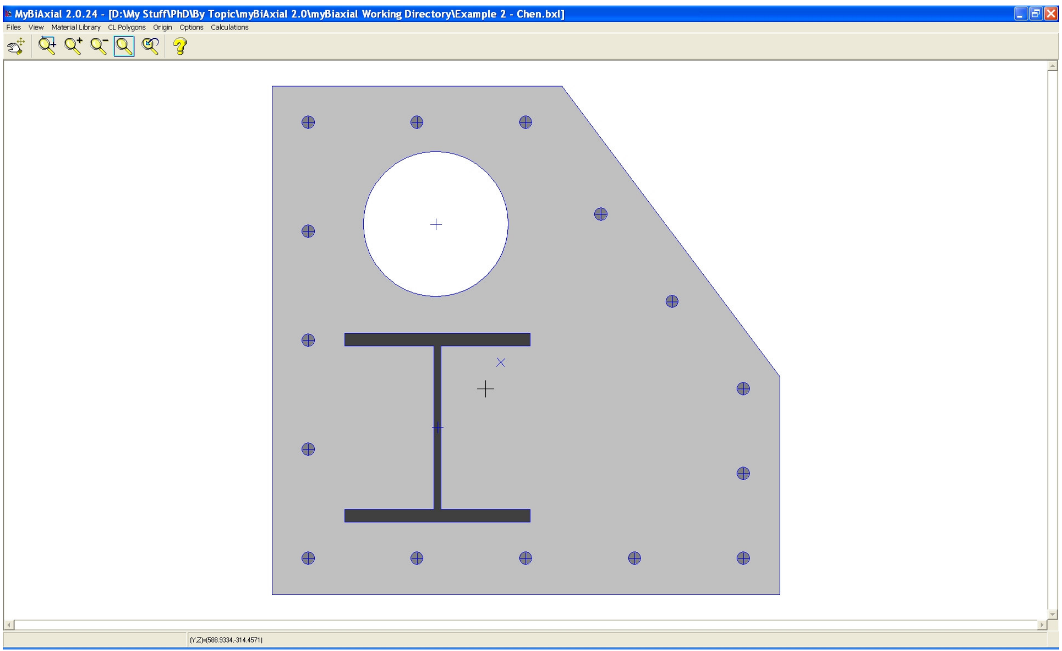


Figure 4. Complete failure surface

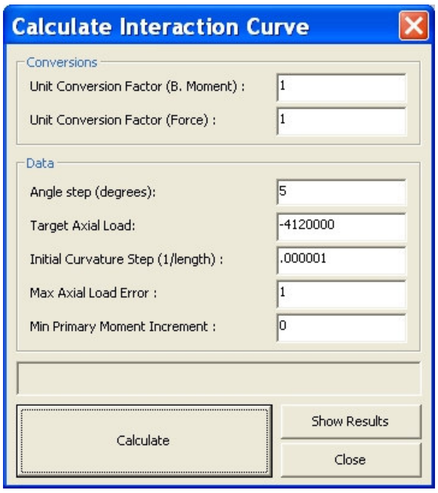
The program setup includes this example and the material data necessary to replicate the results. Select *Files > Load* and select the file *Example 2 - Chen.bxl* which can be found in the application path. The main form will look like this:



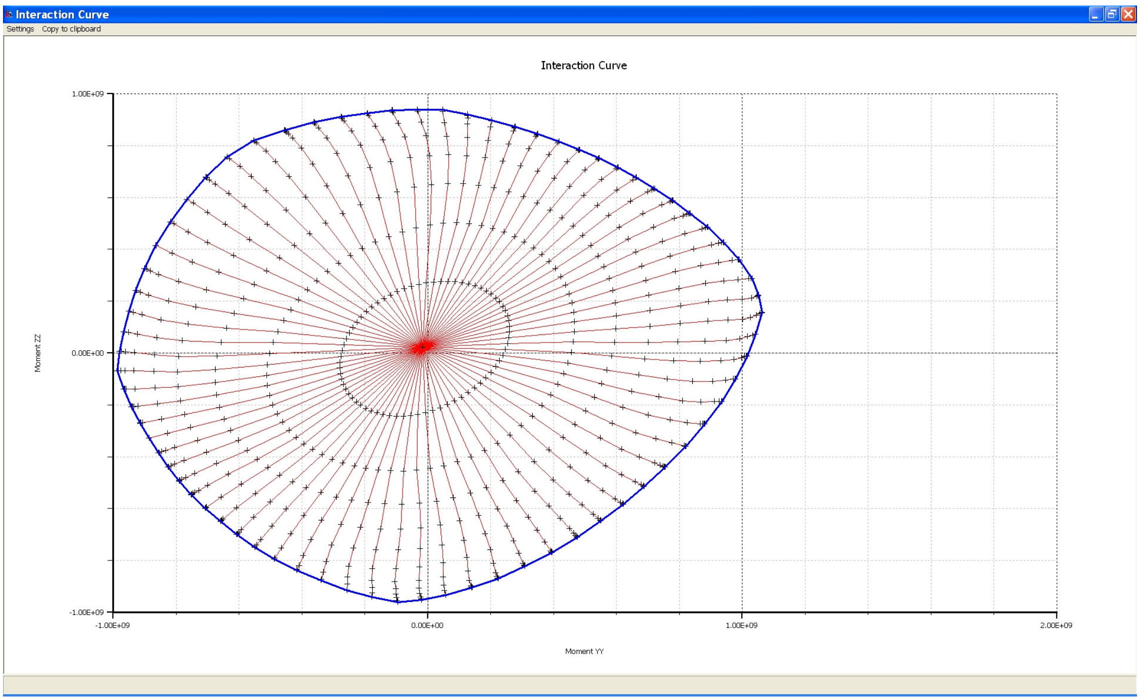
The cross-section was drawn in mm whereas the material stresses were defined in MPa. Therefore the results for force will be in MPa x mm^2 = N whereas the results for moment will be in MPa x mm^3= Nmm.

In order to obtain the results in other units, you should define conversion factors for bending moments and forces. In this case, a conversion factor of 1E-06 and 1E-03 for moments and forces will yield results in kNm and kN respectively.

Select *Calculations > Interaction Curve* . Input the data as shown in the following figure:



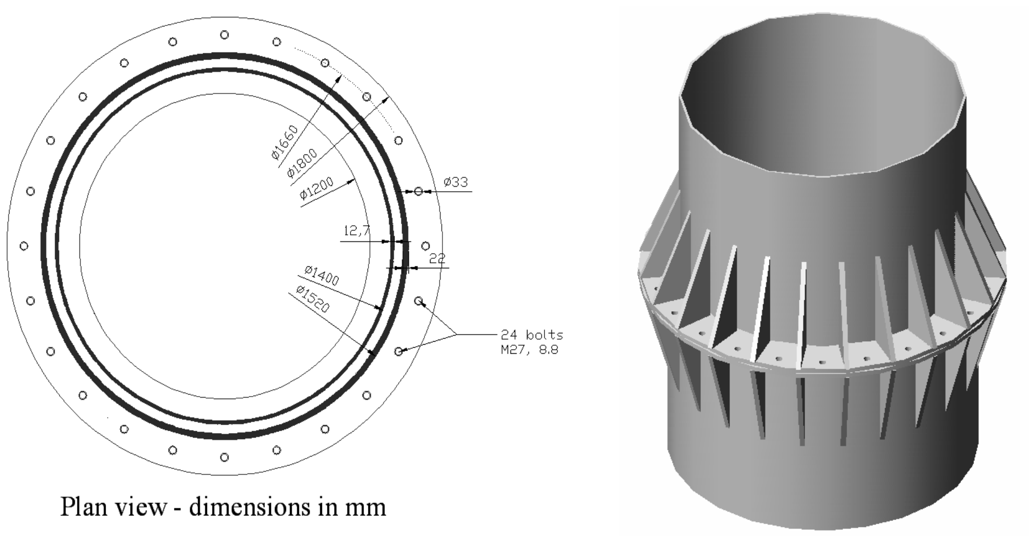
Press *Calculate* to calculate the interaction curve. The curve will look like this:



When scaled properly, this figure is the same as the one in Figure 3. Click on *Show Results* to open the calculation log. This provides information for the intermediate steps as well as additional information for the final step.

4.3 Moment capacity of rigid bolted connection

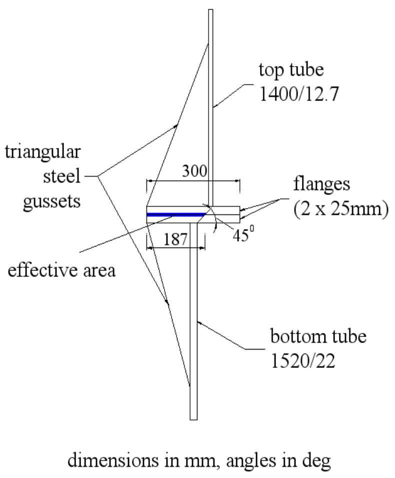
In this example, the versatility of the proposed algorithm is demonstrated. The task is to check the maximum bending moment capacity of a bolted connection of two circular tubes of diameter/width 1520/22mm and 1400/12.7mm respectively. The connection is implemented by means of two circular flanges and 24 bolts arranged in circle. The flanges are reinforced externally by dense out – of – plane triangular steel elements (gussets), as shown in the figures.



(a) (b)

Figure 5. (a) Plan view of the proposed connection
(b) 3D view of the proposed connection

We assume that the flanges are rigid by virtue of the triangular steel elements. However, the rigidity does not extend to the inner circle of the two flanges; we assume that the effective rigid ring has a width of 187mm, as shown in Figure 6a.



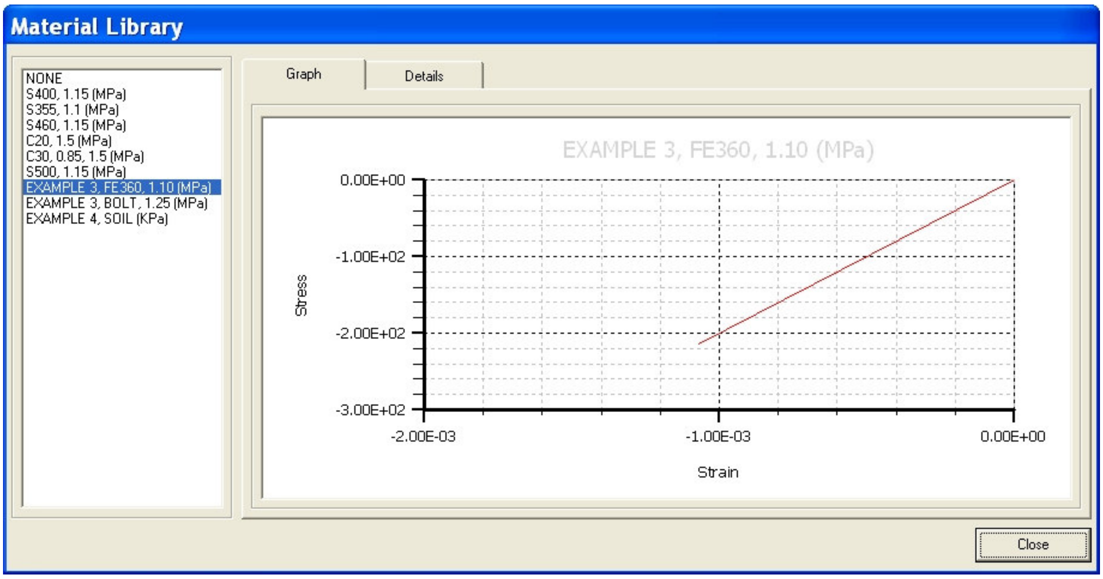
(a)

Property	Value
Bottom tube, external diameter	1520mm
Bottom tube, thickness	22mm
Top tube, external diameter	1400mm
Top tube, thickness	12.7mm
Flange, external diameter	1800mm
Flange, internal diameter	1200mm
Flange, thickness	25mm
Steel grade	S235
Number of bolts	24
Bolt size	M27
Bolt quality	8.8
Bolts arrangement, circle diameter	1660mm
Bolts hole, circle diameter	33mm
Axial load (compressive)	325kN

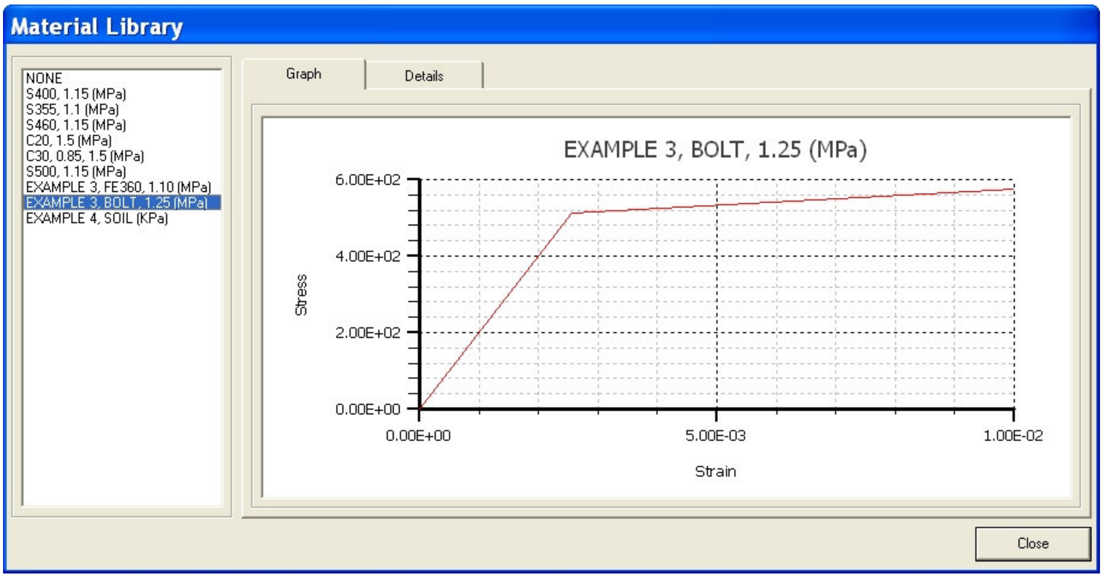
(b)

Figure 6. (a) Section of the proposed connection (b) Table of properties

Two materials are now defined: the flanges behave linearly in compression up to yield strength i.e. 235MPa/1.10=213.636MPa; however they do not exhibit tensile strength. We expect the flanges not to yield i.e. the failure should occur because of the bolts:



We assume that the bolts (quality 8.8) exhibit a bilinear behavior. The first linear segment extends in tension up to yield strength i.e. 640MPa/1.25=512MPa; the second linear segment extends up to ultimate strength defined by EC3, i.e. 0.9·800MPa/1.25=576MPa; however they do not exhibit compressive strength:



Young modulus is taken equal to 200GPa for all cases. Of course, the material properties may be defined otherwise and may also include parabolic or cubic segments, subject to the user's needs or assumptions.

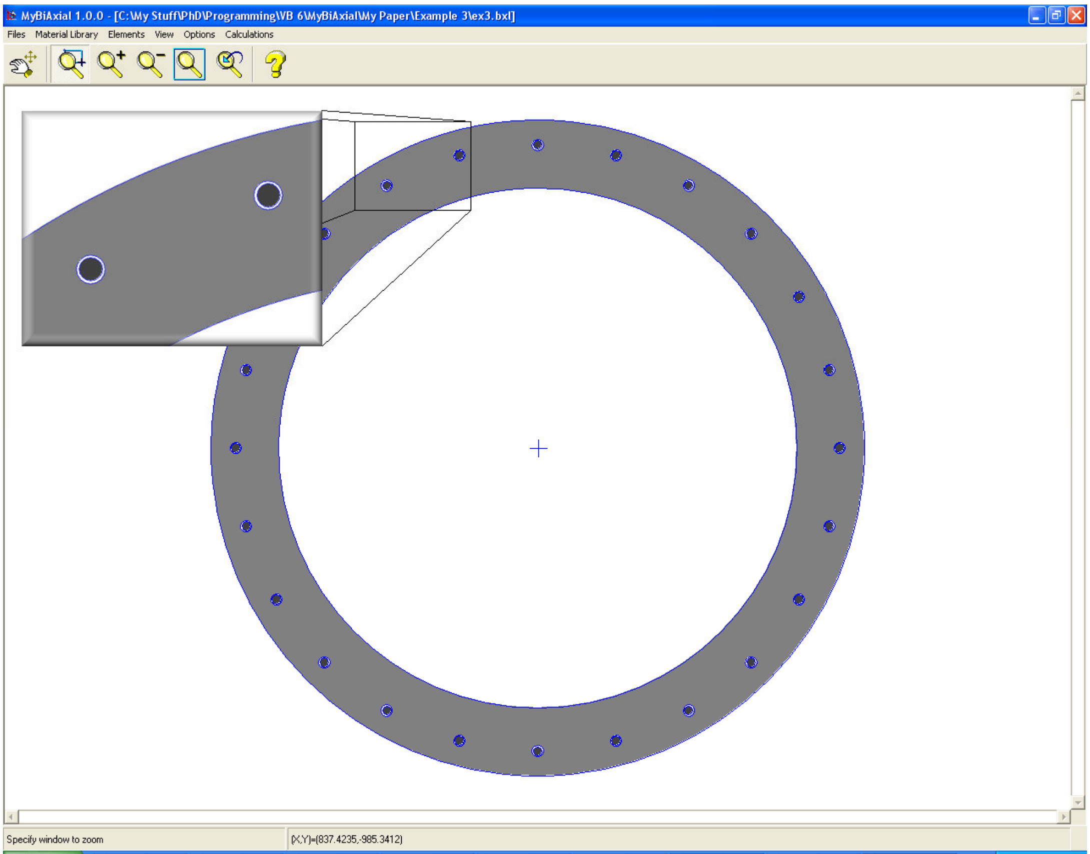


Figure 7. Example 3

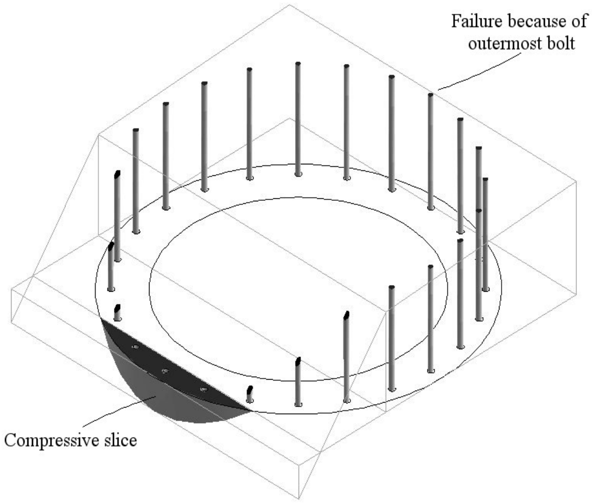


Figure 8. 3D view of stress solids - verification of results using CAD software

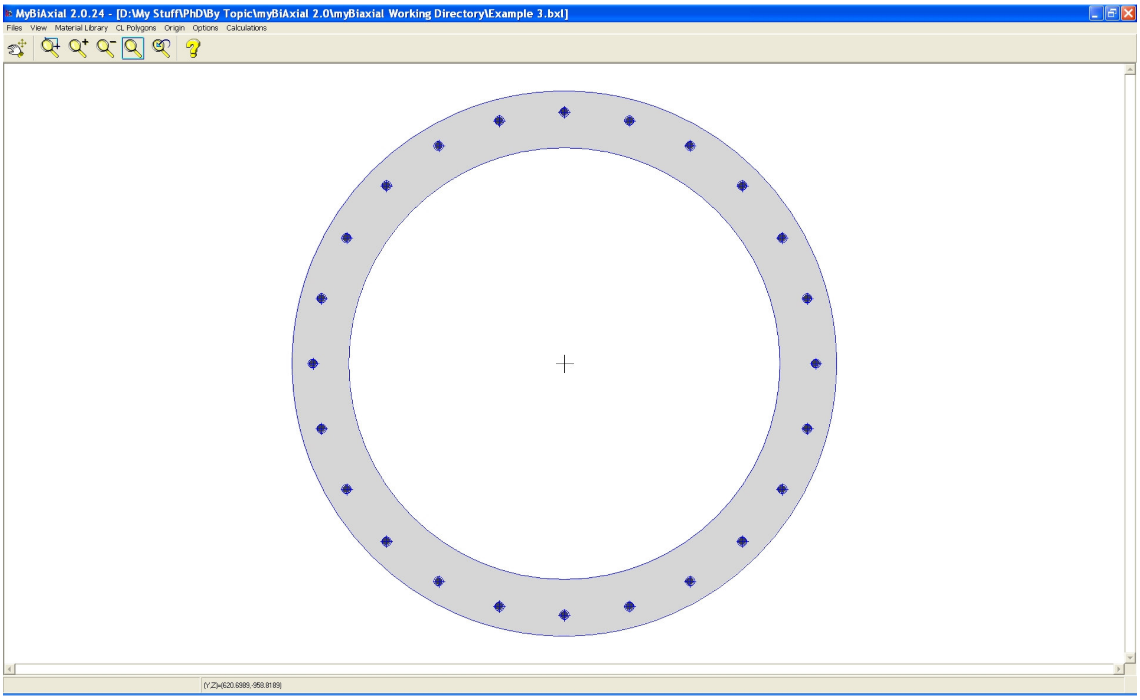
For an axial (compressive) load of N_{Xc}=325kN, the algorithm yields the following results: curvature φ=6.223·10-6 mm^{-1}, strain at the origin ε_0=4.751·10-3, ultimate bending moment at failure M_{Yc}=6466.160kNm. The minimum strain for the flanges is $\varepsilon_{min,flanges}$=-8.493·10-4; therefore, the flanges do not yield, as assumed from the beginning. The failure occurs because of the outermost bolt, which reaches the maximum strain of $\varepsilon_{max,bolts}$=±0.010.

Based on these data, the stress solids were created using CAD software (Figure 8). The results are summarized in Table 9; the sum of the volume of all stress solids is equal to the axial load and the sum of all moments is equal to the result obtained from the proposed algorithm.

Element	Volume (or Force, kN)	Y_c Coordinate of Centroid (mm)	Bending Moment M_{Yc} (kNm)
Flange	-5876.257	-842.452	4950.463
Bolts #11 (· 2)	63.718	-717.782	-45.735
Bolts #10 (· 2)	251.698	-586.641	-147.656
Bolts #9 (· 2)	496.678	-414.869	-206.056
Bolts #8 (· 2)	594.712	-214.815	-127.753
Bolts #7 (· 2)	607.880	0.000	0.000
Bolts #6 (· 2)	621.048	214.824	133.416
Bolts #5 (· 2)	633.318	415.004	262.830
Bolts #4 (· 2)	643.855	586.903	377.880
Bolts #3 (· 2)	651.940	718.805	468.618
Bolts #2 (· 2)	657.023	801.723	526.750
Bolt #1 (· 1)	329.378	830.004	273.385
Sums :	-325.009		6466.141

Table 9. Computed results from CAD software

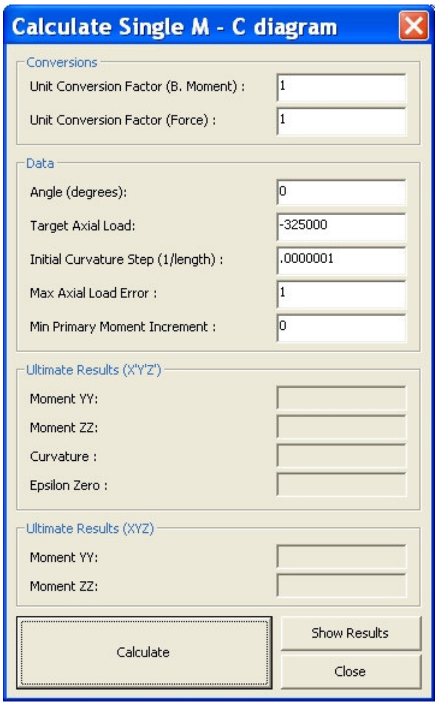
The program setup includes this example and the material data necessary to replicate the results. Select *Files > Load* and select the file *Example 3.bxl* which can be found in the application path. The main form will look like this:



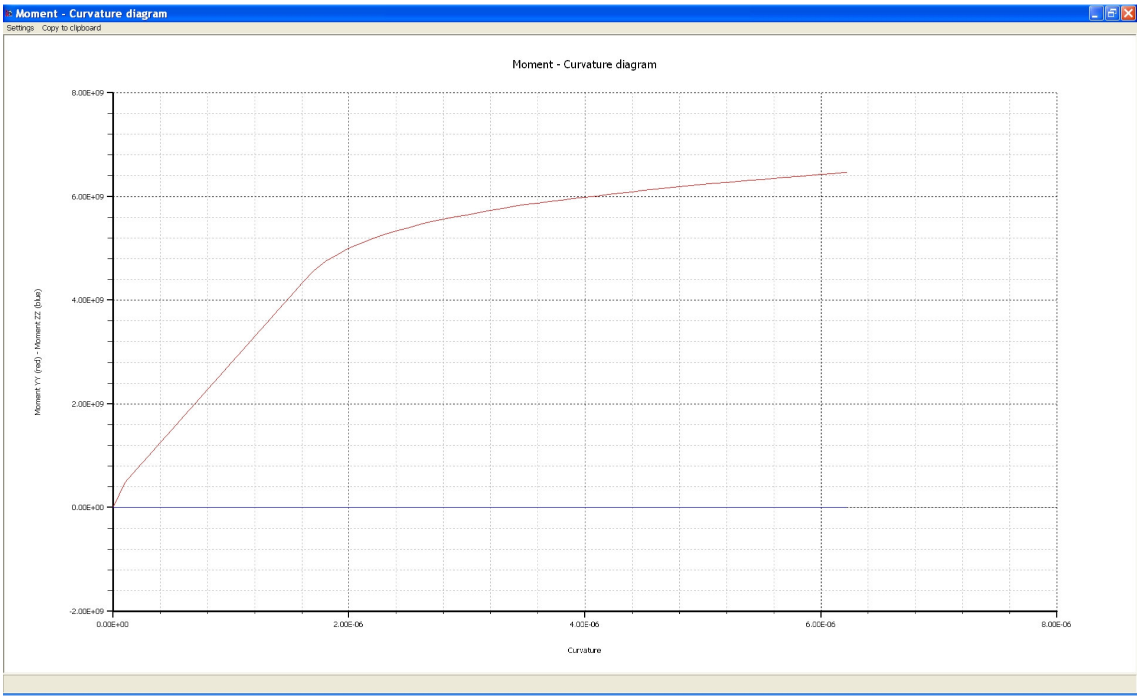
The cross-section was drawn in mm whereas the material stresses were defined in MPa. Therefore the results for force will be in MPa x mm^2 = N whereas the results for moment will be in MPa x mm^3= Nmm.

In order to obtain the results in other units, you should define conversion factors for bending moments and forces. In this case, a conversion factor of 1E-06 and 1E-03 for moments and forces will yield results in kNm and kN respectively.

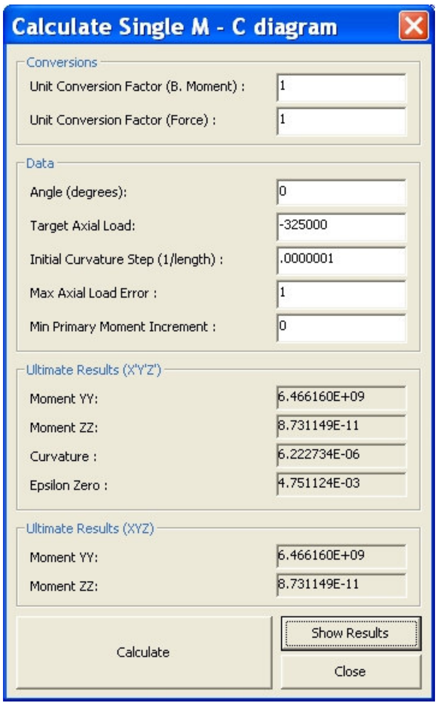
Select *Calculations > Single Moment – Curvature Diagram*. Input the data as shown in the following figure:



Press *Calculate* to calculate the moment – curvature diagram. The moment – curvature diagram will look like this:



The ultimate results are shown in the form:



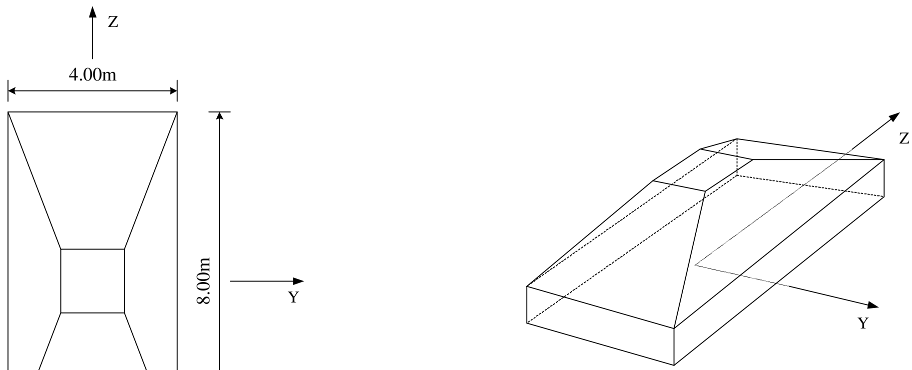
Click on *Show Results* to open the calculation log. This provides information for the intermediate steps as well as additional information for the final step. For this case, excerpt of the calculation log with the strains in the final step is shown in the following table. Note the highlighted strain value of 1.000E-2 at the final step of the outermost bolt.

```
Strains at final step:
# CLTrapezoid       Foreground Material Background Material Z bottom           Z top              Min Strain         Max Strain
1                   EXAMPLE 3, FE360,   NONE                -900,000000000     900,000000000      -8,493361856E-04   1,035158445E-02
2                   NONE                EXAMPLE 3, FE360,   -16,500000000      16,500000000       4,648449028E-03    4,853799240E-03
3                   NONE                EXAMPLE 3, FE360,   198,319807000      231,319807000      5,985215478E-03    6,190565690E-03
4                   NONE                EXAMPLE 3, FE360,   398,500000000      431,500000000      7,230883509E-03    7,436233720E-03
5                   NONE                EXAMPLE 3, FE360,   570,398628000      603,398628000      8,300562892E-03    8,505913104E-03
6                   NONE                EXAMPLE 3, FE360,   702,301085000      735,301085000      9,121356755E-03    9,326706967E-03
7                   NONE                EXAMPLE 3, FE360,   785,218436000      818,218436000      9,637329348E-03    9,842679560E-03
8                   NONE                EXAMPLE 3, FE360,   813,500000000      846,500000000      9,813317989E-03    1,001866820E-02
9                   NONE                EXAMPLE 3, FE360,   785,218436000      818,218436000      9,637329348E-03    9,842679560E-03
10                  NONE                EXAMPLE 3, FE360,   702,301085000      735,301085000      9,121356755E-03    9,326706967E-03
11                  NONE                EXAMPLE 3, FE360,   570,398628000      603,398628000      8,300562892E-03    8,505913104E-03
12                  NONE                EXAMPLE 3, FE360,   398,500000000      431,500000000      7,230883509E-03    7,436233720E-03
13                  NONE                EXAMPLE 3, FE360,   198,319807000      231,319807000      5,985215478E-03    6,190565690E-03
14                  NONE                EXAMPLE 3, FE360,   -16,500000000      16,500000000       4,648449028E-03    4,853799240E-03
15                  NONE                EXAMPLE 3, FE360,   -231,319807000     -198,319807000     3,311682578E-03    3,517032790E-03
16                  NONE                EXAMPLE 3, FE360,   -431,500000000     -398,500000000     2,066014547E-03    2,271364759E-03
17                  NONE                EXAMPLE 3, FE360,   -603,398628000     -570,398628000     9,963351639E-04    1,201685376E-03
18                  NONE                EXAMPLE 3, FE360,   -735,301085000     -702,301085000     1,755413012E-04    3,808915129E-04
19                  NONE                EXAMPLE 3, FE360,   -818,218436000     -785,218436000     -3,404312923E-04   -1,350810805E-04
20                  NONE                EXAMPLE 3, FE360,   -846,500000000     -813,500000000     -5,164199333E-04   -3,110697216E-04
21                  NONE                EXAMPLE 3, FE360,   -818,218436000     -785,218436000     -3,404312923E-04   -1,350810805E-04
22                  NONE                EXAMPLE 3, FE360,   -735,301085000     -702,301085000     1,755413012E-04    3,808915129E-04
23                  NONE                EXAMPLE 3, FE360,   -603,398628000     -570,398628000     9,963351639E-04    1,201685376E-03
24                  NONE                EXAMPLE 3, FE360,   -431,500000000     -398,500000000     2,066014547E-03    2,271364759E-03
25                  NONE                EXAMPLE 3, FE360,   -231,319807000     -198,319807000     3,311682578E-03    3,517032790E-03
26                  EXAMPLE 3, BOLT, 1  NONE                -13,500000000      13,500000000       4,667117229E-03    4,835131039E-03
27                  EXAMPLE 3, BOLT, 1  NONE                201,319807000      228,319807000      6,003883679E-03    6,171897489E-03
28                  EXAMPLE 3, BOLT, 1  NONE                401,500000000      428,500000000      7,249551710E-03    7,417565519E-03
29                  EXAMPLE 3, BOLT, 1  NONE                573,398628000      600,398628000      8,319231093E-03    8,487244903E-03
30                  EXAMPLE 3, BOLT, 1  NONE                705,301085000      732,301085000      9,140024956E-03    9,308038766E-03
31                  EXAMPLE 3, BOLT, 1  NONE                788,218436000      815,218436000      9,655997549E-03    9,824011359E-03
32                  EXAMPLE 3, BOLT, 1  NONE                816,500000000      843,500000000      9,831986190E-03    1,000000000E-02
33                  EXAMPLE 3, BOLT, 1  NONE                788,218436000      815,218436000      9,655997549E-03    9,824011359E-03
34                  EXAMPLE 3, BOLT, 1  NONE                705,301085000      732,301085000      9,140024956E-03    9,308038766E-03
35                  EXAMPLE 3, BOLT, 1  NONE                573,398628000      600,398628000      8,319231093E-03    8,487244903E-03
36                  EXAMPLE 3, BOLT, 1  NONE                401,500000000      428,500000000      7,249551710E-03    7,417565519E-03
37                  EXAMPLE 3, BOLT, 1  NONE                201,319807000      228,319807000      6,003883679E-03    6,171897489E-03
38                  EXAMPLE 3, BOLT, 1  NONE                -13,500000000      13,500000000       4,667117229E-03    4,835131039E-03
39                  EXAMPLE 3, BOLT, 1  NONE                -228,319807000     -201,319807000     3,330350779E-03    3,498364589E-03
40                  EXAMPLE 3, BOLT, 1  NONE                -428,500000000     -401,500000000     2,084682748E-03    2,252696558E-03
41                  EXAMPLE 3, BOLT, 1  NONE                -600,398628000     -573,398628000     1,015003365E-03    1,183017175E-03
42                  EXAMPLE 3, BOLT, 1  NONE                -732,301085000     -705,301085000     1,942095022E-04    3,622233118E-04
43                  EXAMPLE 3, BOLT, 1  NONE                -815,218436000     -788,218436000     -3,217630912E-04   -1,537492816E-04
44                  EXAMPLE 3, BOLT, 1  NONE                -843,500000000     -816,500000000     -4,977517323E-04   -3,297379227E-04
45                  EXAMPLE 3, BOLT, 1  NONE                -815,218436000     -788,218436000     -3,217630912E-04   -1,537492816E-04
46                  EXAMPLE 3, BOLT, 1  NONE                -732,301085000     -705,301085000     1,942095022E-04    3,622233118E-04
47                  EXAMPLE 3, BOLT, 1  NONE                -600,398628000     -573,398628000     1,015003365E-03    1,183017175E-03
48                  EXAMPLE 3, BOLT, 1  NONE                -428,500000000     -401,500000000     2,084682748E-03    2,252696558E-03
49                  EXAMPLE 3, BOLT, 1  NONE                -228,319807000     -201,319807000     3,330350779E-03    3,498364589E-03
50                  NONE                EXAMPLE 3, FE360,   -713,000000000     713,000000000      3,143150141E-04    9,187933254E-03
```

Table 10. Example 3: Excerpt of calculation log

4.4 Rigid footing

In this example, the task is to calculate the maximum bending moment capacity of a rigid footing (Figure 9). We assume that the footing is placed over sand modeled with independent springs (Winkler); failure occurs when stress exceeds a predefined maximum value.



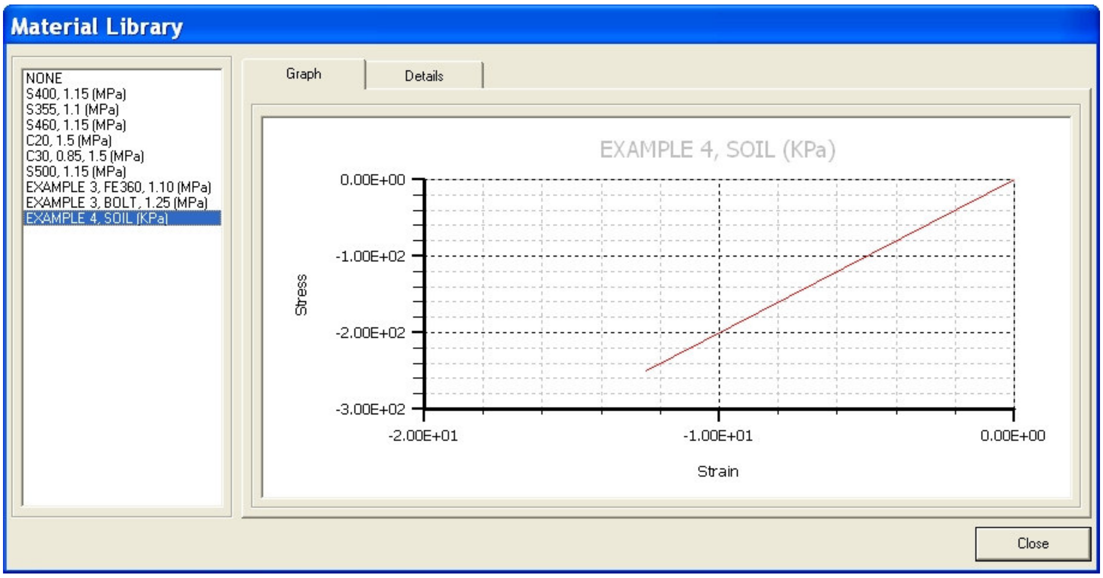
(a) (b)

Figure 9. (a) Plan view of rigid footing (b) 3D view of rigid footing

Property	Value
Rigid footing, length	8.00m
Rigid footing, width	4.00m
Axial load (compressive)	1300kN
Sand, k	20kPa/mm
Sand, maximum stress	250kPa

Table 11. Properties

For the material properties, instead of using curvature and strain we will be using slope and settlement. We assume that sand behaves linearly in compression up to a maximum stress of 250kPa with a subgrade modulus k_s=20kPa/mm (maximum settlement 12.5mm); also, it does not exhibit tensile strength. Note that linear behavior is not obligatory; moreover, in this case, the stresses are expressed with respect to settlement instead of strain:



For an axial (compressive) load of N_{XC}=1300kN, the algorithm yields the following results: slope k=4.808, settlement at the origin ε_0=6.731mm, ultimate bending moment at failure M_{YC}=4073.331kNm. The failure occurs because the sand reaches the maximum stress capacity of 250kPa (Figure 10a). The results are easily verifiable (equation (3))

$$\begin{aligned} N &= \frac{1}{2}\cdot 250KPa\cdot 2.60m\cdot 4.00m = 1300kN \\ M &= 1300kN\cdot\left(1.40m+\frac{2}{3}\cdot 2.60m\right) = 4073.333kNm \end{aligned} \tag{3}$$

As a step further, we may want to restrict the length of the ineffective area of the footing. This is achieved easily by applying a restriction similar to that of "Point C" of EC 2, which is described in Example 1. For example, we demand that the settlement at distance ½ h from the most compressed point i.e. at the middle of the footing, to be less than or equal to zero. In this way, more than half of the footing is always in contact with the sand. In this case and for the same axial (compressive) load of N_{XC}=1300kN, the algorithm yields the following results: slope k=2.031, settlement at the origin ε_0=0.000mm, ultimate bending moment at failure M_{YC}=3466.667kNm (Figure 10b). Again, the results are easily verifiable (equation (4):

$$\begin{aligned} N &= \frac{1}{2}\cdot 162.5KPa\cdot 4.00m\cdot 4.00m = 1300kN \\ M &= 1300kN\cdot\left(\frac{2}{3}\cdot 4.00m\right) = 3466.666kNm \end{aligned} \tag{4}$$

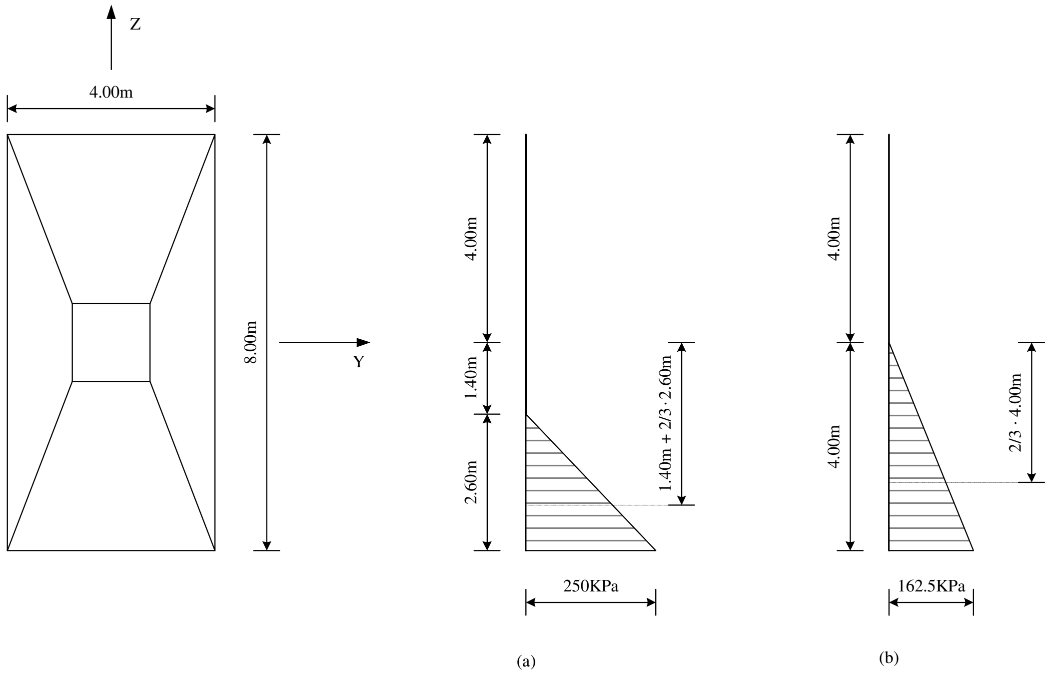
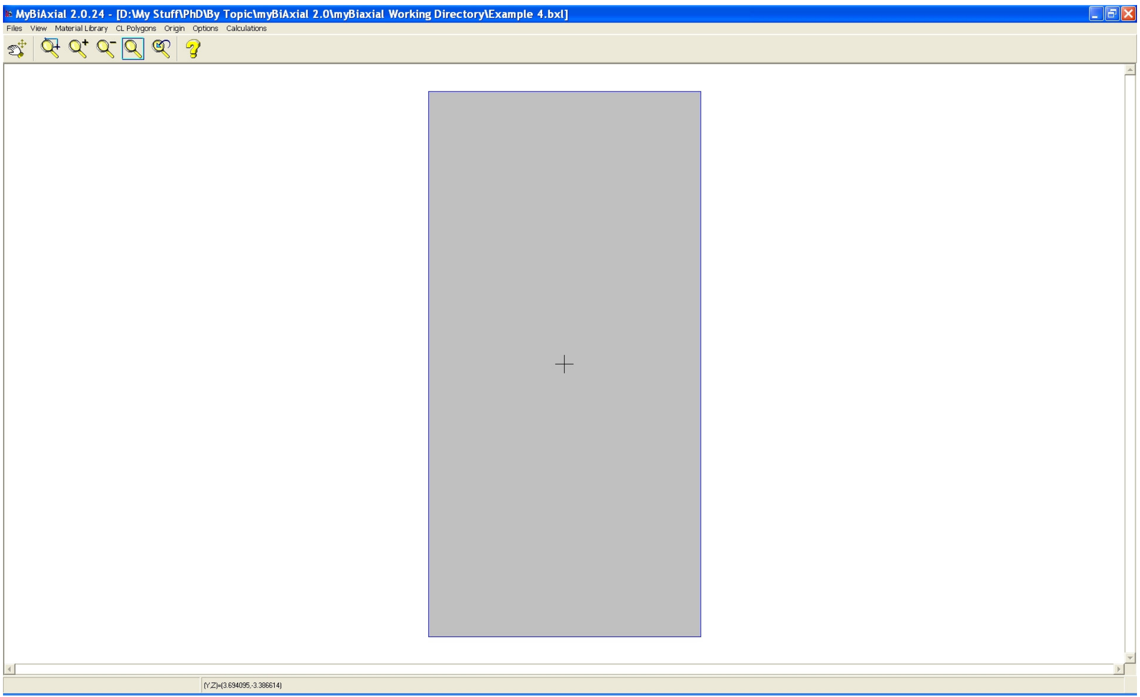


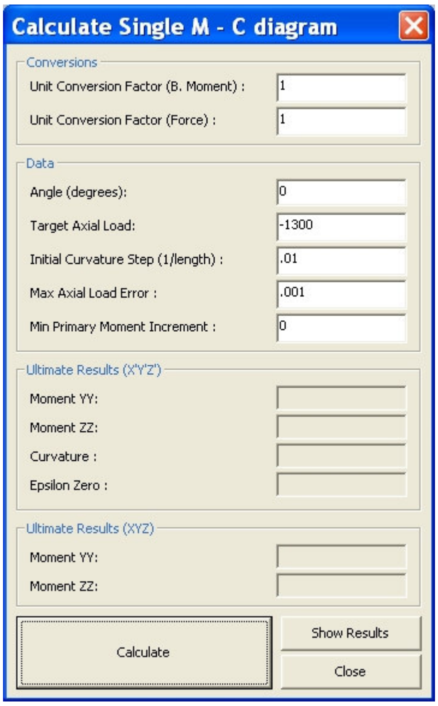
Figure 10. (a) Stresses with no restriction (compressive axial load 1300kN)
(b) Stresses with restriction at midpoint of rigid footing (compressive axial load 1300kN)

The program setup includes this example and the material data necessary to replicate the results. Select *Files > Load* and select the file *Example 4.bxl* which can be found in the application path. The main form will look like this:

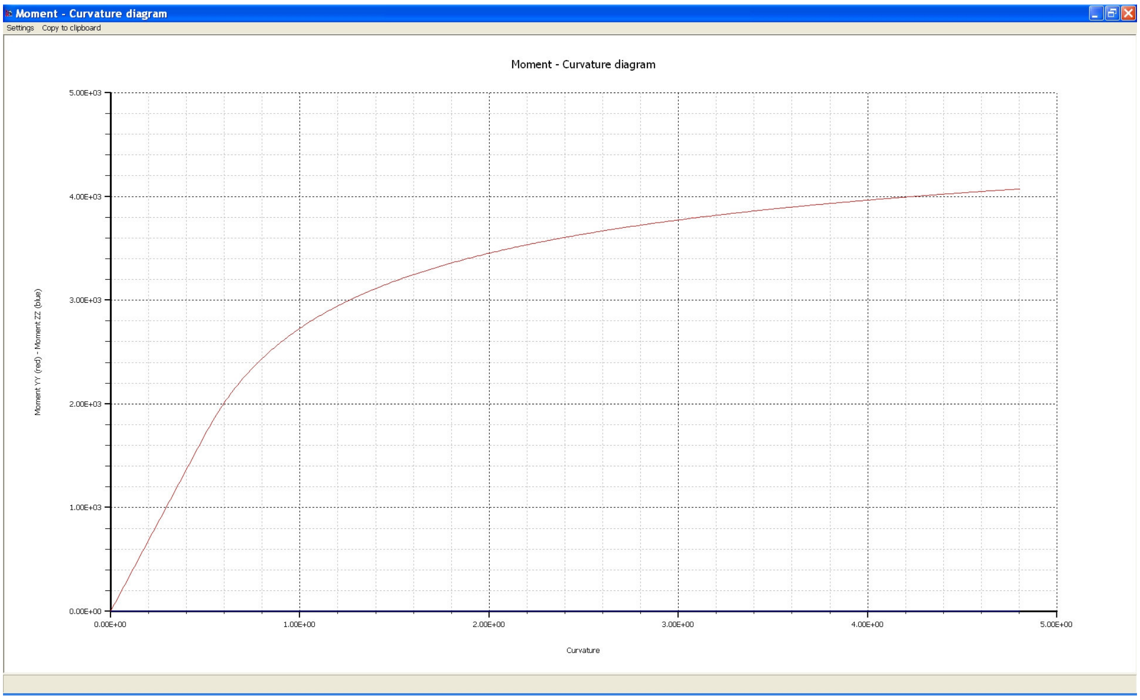


The cross-section was drawn in m whereas the material stresses were defined in KPa. Therefore the results for force will be in KPa x m^2 = kN whereas the results for moment will be in KPa x m^3= kNm. The material "strains" were defined in mm.

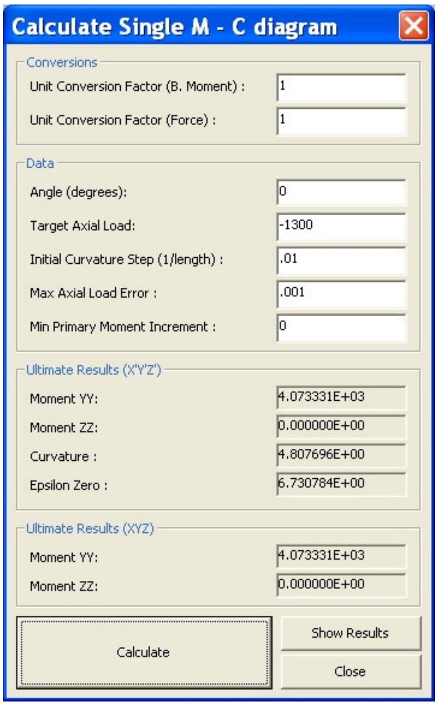
Select *Calculations > Single Moment – Curvature Diagram* . Input the data as shown in the following figure:



Press *Calculate* to calculate the moment – curvature diagram. The diagram will look like this:



The ultimate results are shown in the form:

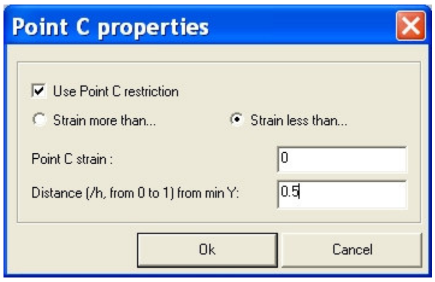


Click on *Show Results* to open the calculation log. This provides information for the intermediate steps as well as additional information for the final step. For this case, the calculation log is shown in the following table. Note the highlighted settlement value of -1.250E+01 at the final step.

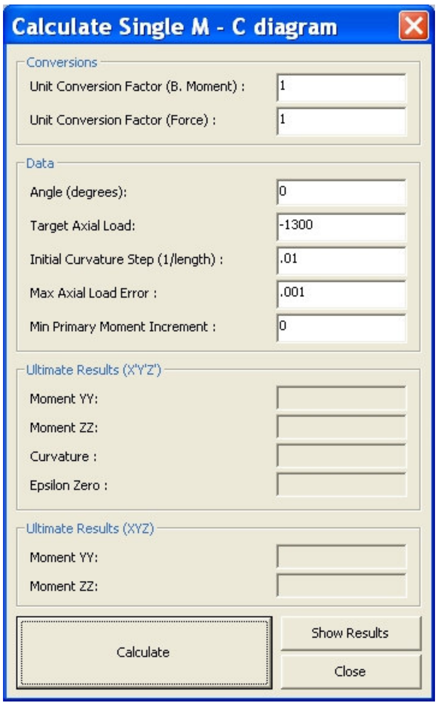
```
…
444               5,721041712E+00    4,430000000E+00    -1,300000000E+03    4,026287000E+03    0,000000000E+00    6
445               5,747506504E+00    4,440000000E+00    -1,300000000E+03    4,027609494E+03    0,000000000E+00    6
446               5,773986529E+00    4,450000000E+00    -1,300000000E+03    4,028927527E+03    0,000000000E+00    6
447               5,800481737E+00    4,460000000E+00    -1,300000000E+03    4,030241125E+03    0,000000000E+00    6
448               5,826992077E+00    4,470000000E+00    -1,300000000E+03    4,031550313E+03    0,000000000E+00    6
449               5,853517497E+00    4,480000000E+00    -1,300000000E+03    4,032855115E+03    0,000000000E+00    6
450               5,880057947E+00    4,490000000E+00    -1,300000000E+03    4,034155555E+03    0,000000000E+00    6
451               5,906613378E+00    4,500000000E+00    -1,300000000E+03    4,035451659E+03    0,000000000E+00    6
452               5,933183738E+00    4,510000000E+00    -1,300000000E+03    4,036743449E+03    0,000000000E+00    6
453               5,959768979E+00    4,520000000E+00    -1,300000000E+03    4,038030949E+03    0,000000000E+00    6
454               5,986369051E+00    4,530000000E+00    -1,300000000E+03    4,039314184E+03    0,000000000E+00    6
455               6,012983905E+00    4,540000000E+00    -1,300000000E+03    4,040593177E+03    0,000000000E+00    6
456               6,039613493E+00    4,550000000E+00    -1,300000000E+03    4,041867952E+03    0,000000000E+00    6
457               6,066257765E+00    4,560000000E+00    -1,300000000E+03    4,043138530E+03    0,000000000E+00    6
458               6,092916674E+00    4,570000000E+00    -1,300000000E+03    4,044404936E+03    0,000000000E+00    6
459               6,119590171E+00    4,580000000E+00    -1,300000000E+03    4,045667192E+03    0,000000000E+00    6
460               6,146278209E+00    4,590000000E+00    -1,300000000E+03    4,046925321E+03    0,000000000E+00    6
461               6,172980739E+00    4,600000000E+00    -1,300000000E+03    4,048179345E+03    0,000000000E+00    6
462               6,199697716E+00    4,610000000E+00    -1,300000000E+03    4,049429286E+03    0,000000000E+00    6
463               6,226429092E+00    4,620000000E+00    -1,300000000E+03    4,050675167E+03    0,000000000E+00    6
464               6,253174820E+00    4,630000000E+00    -1,300000000E+03    4,051917010E+03    0,000000000E+00    6
465               6,279934854E+00    4,640000000E+00    -1,300000000E+03    4,053154835E+03    0,000000000E+00    6
466               6,306709147E+00    4,650000000E+00    -1,300000000E+03    4,054388666E+03    0,000000000E+00    6
467               6,333497654E+00    4,660000000E+00    -1,300000000E+03    4,055618523E+03    0,000000000E+00    6
468               6,360300328E+00    4,670000000E+00    -1,300000000E+03    4,056844427E+03    0,000000000E+00    6
469               6,387117125E+00    4,680000000E+00    -1,300000000E+03    4,058066400E+03    0,000000000E+00    6
470               6,413947999E+00    4,690000000E+00    -1,300000000E+03    4,059284463E+03    0,000000000E+00    6
471               6,440792906E+00    4,700000000E+00    -1,300000000E+03    4,060498637E+03    0,000000000E+00    6
472               6,467651799E+00    4,710000000E+00    -1,300000000E+03    4,061708941E+03    0,000000000E+00    6
473               6,494524636E+00    4,720000000E+00    -1,300000000E+03    4,062915397E+03    0,000000000E+00    6
474               6,521411371E+00    4,730000000E+00    -1,300000000E+03    4,064118026E+03    0,000000000E+00    6
475               6,548311960E+00    4,740000000E+00    -1,300000000E+03    4,065316846E+03    0,000000000E+00    6
476               6,575226360E+00    4,750000000E+00    -1,300000000E+03    4,066511878E+03    0,000000000E+00    6
477               6,602154527E+00    4,760000000E+00    -1,300000000E+03    4,067703143E+03    0,000000000E+00    6
478               6,629096418E+00    4,770000000E+00    -1,300000000E+03    4,068890660E+03    0,000000000E+00    6
479               6,656054610E+00    4,780000000E+00    -1,299999453E+03    4,070072974E+03    0,000000000E+00    5
480               6,683021199E+00    4,790000000E+00    -1,300000000E+03    4,071254527E+03    0,000000000E+00    6
481               6,710004205E+00    4,800000000E+00    -1,299999958E+03    4,072430804E+03    0,000000000E+00    5
482               6,723500980E+00    4,805000000E+00    -1,299999898E+03    4,073017459E+03    0,000000000E+00    5
483               6,730250005E+00    4,807500000E+00    -1,299999999E+03    4,073310798E+03    0,000000000E+00    5
484               6,730671875E+00    4,807656250E+00    -1,300000000E+03    4,073329108E+03    0,000000000E+00    5
485               6,730781250E+00    4,807695312E+00    -1,299999188E+03    4,073331492E+03    0,000000000E+00    3
486               6,730783691E+00    4,807695923E+00    -1,299999022E+03    4,073331118E+03    0,000000000E+00    3

Strains at final step:

# CLTrapezoid       Foreground Material Background Material Z bottom           Z top              Min Strain         Max Strain
1                   EXAMPLE 4, SOIL (K  NONE                -4,000000000       4,000000000        -1,250000000E+01   2,596156738E+01
```

Table 12. Example 4: Excerpt of calculation log (case 1)

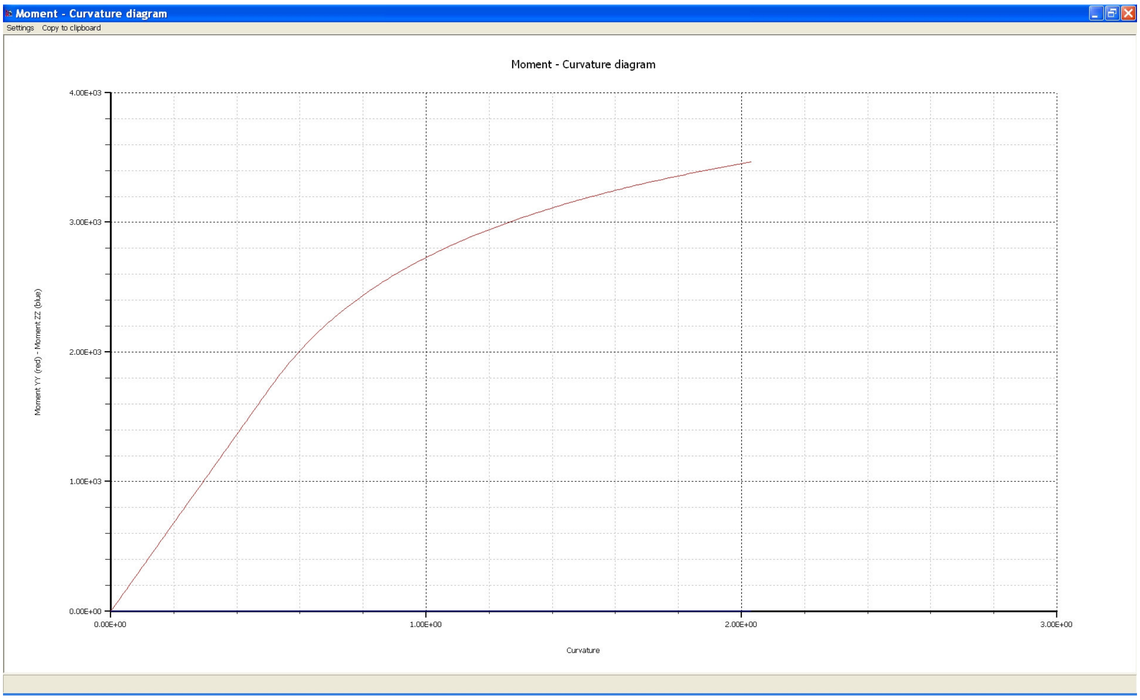
For the second case, select *Calculations > Point C Restrictions* and input the data as follows:



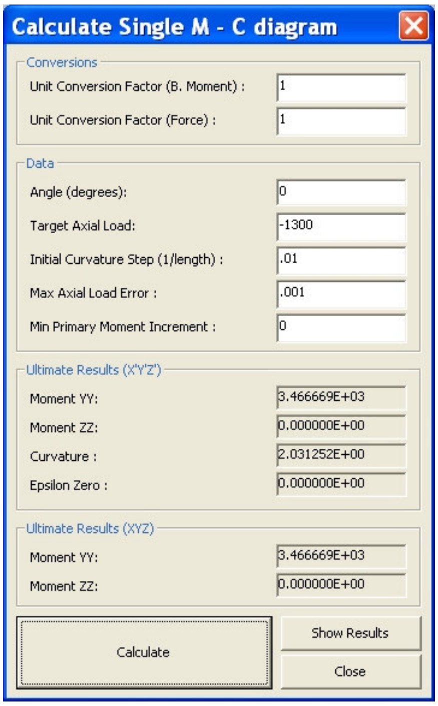
Select *Calculations > Single Moment – Curvature Diagram*. Input the data as shown in the following figure:



Press *Calculate* to calculate the moment – curvature diagram. The diagram will look like this:



The ultimate results are shown in the form:



Click on *Show Results* to open the calculation log. This provides information for the intermediate steps as well as additional information for the final step. For this case, the calculation log is shown in the following table. Note the highlighted top and bottom settlements of equal magnitude at the final step.

```
…
185    -3,730436927E-01    1,840000000E+00    -1,299999234E+03    3,378810182E+03    0,000000000E+00    5
186    -3,540290483E-01    1,850000000E+00    -1,299999275E+03    3,383739057E+03    0,000000000E+00    5
187    -3,349577636E-01    1,860000000E+00    -1,299999317E+03    3,388628131E+03    0,000000000E+00    5
188    -3,158302946E-01    1,870000000E+00    -1,299999358E+03    3,393477935E+03    0,000000000E+00    5
189    -2,966470914E-01    1,880000000E+00    -1,299999399E+03    3,398288993E+03    0,000000000E+00    5
190    -2,774085982E-01    1,890000000E+00    -1,299999440E+03    3,403061818E+03    0,000000000E+00    5
191    -2,581152532E-01    1,900000000E+00    -1,299999481E+03    3,407796914E+03    0,000000000E+00    5
192    -2,387674887E-01    1,910000000E+00    -1,299999521E+03    3,412494776E+03    0,000000000E+00    5
193    -2,193657317E-01    1,920000000E+00    -1,299999562E+03    3,417155888E+03    0,000000000E+00    5
194    -1,999104033E-01    1,930000000E+00    -1,299999602E+03    3,421780727E+03    0,000000000E+00    5
195    -1,804019194E-01    1,940000000E+00    -1,299999642E+03    3,426369762E+03    0,000000000E+00    5
196    -1,608406905E-01    1,950000000E+00    -1,299999682E+03    3,430923452E+03    0,000000000E+00    5
197    -1,412271217E-01    1,960000000E+00    -1,299999722E+03    3,435442248E+03    0,000000000E+00    5
198    -1,215616131E-01    1,970000000E+00    -1,299999761E+03    3,439926594E+03    0,000000000E+00    5
199    -1,018445597E-01    1,980000000E+00    -1,299999801E+03    3,444376926E+03    0,000000000E+00    5
200    -8,207635145E-02    1,990000000E+00    -1,299999840E+03    3,448793670E+03    0,000000000E+00    5
201    -6,225737344E-02    2,000000000E+00    -1,299999879E+03    3,453177248E+03    0,000000000E+00    5
202    -4,238800596E-02    2,010000000E+00    -1,299999918E+03    3,457528072E+03    0,000000000E+00    5
203    -2,246875996E-02    2,020000000E+00    -1,300000000E+03    3,461846636E+03    0,000000000E+00    5
204    -2,499615271E-03    2,030000000E+00    -1,300000000E+03    3,466133087E+03    0,000000000E+00    5
205    0,000000000E+00     2,031250000E+00    -1,300000000E+03    3,466666667E+03    0,000000000E+00    4
206    0,000000000E+00     2,031251221E+00    -1,300000781E+03    3,466668750E+03    0,000000000E+00    4
207    0,000000000E+00     2,031251526E+00    -1,300000977E+03    3,466669271E+03    0,000000000E+00    4
208    0,000000000E+00     2,031251545E+00    -1,300000989E+03    3,466669303E+03    0,000000000E+00    4
209    0,000000000E+00     2,031251554E+00    -1,300000995E+03    3,466669320E+03    0,000000000E+00    4
210    0,000000000E+00     2,031251559E+00    -1,300000998E+03    3,466669328E+03    0,000000000E+00    4
211    0,000000000E+00     2,031251562E+00    -1,300000999E+03    3,466669332E+03    0,000000000E+00    4
212    0,000000000E+00     2,031251562E+00    -1,300001000E+03    3,466669333E+03    0,000000000E+00    4
213    0,000000000E+00     2,031251562E+00    -1,300001000E+03    3,466669333E+03    0,000000000E+00    4
214    0,000000000E+00     2,031251562E+00    -1,300001000E+03    3,466669333E+03    0,000000000E+00    4
215    0,000000000E+00     2,031251562E+00    -1,300001000E+03    3,466669333E+03    0,000000000E+00    4
216    0,000000000E+00     2,031251562E+00    -1,300001000E+03    3,466669333E+03    0,000000000E+00    4
217    0,000000000E+00     2,031251562E+00    -1,300001000E+03    3,466669333E+03    0,000000000E+00    4
218    0,000000000E+00     2,031251562E+00    -1,300001000E+03    3,466669333E+03    0,000000000E+00    4
219    0,000000000E+00     2,031251562E+00    -1,300001000E+03    3,466669333E+03    0,000000000E+00    4

Strains at final step:

# CLTrapezoid    Foreground Material    Background Material    Z bottom       Z top          Min Strain           Max Strain
1                EXAMPLE 4, SOIL (K     NONE                   -4,000000000   4,000000000    -8,125006250E+00     8,125006250E+00
```

Table 13. Example 4: Excerpt of calculation log (case 2)

We can also create interaction curves by selecting *Calculations > Interaction Curve* . If we include the contact restriction at midpoint, the interaction curve is shown in the following figure:

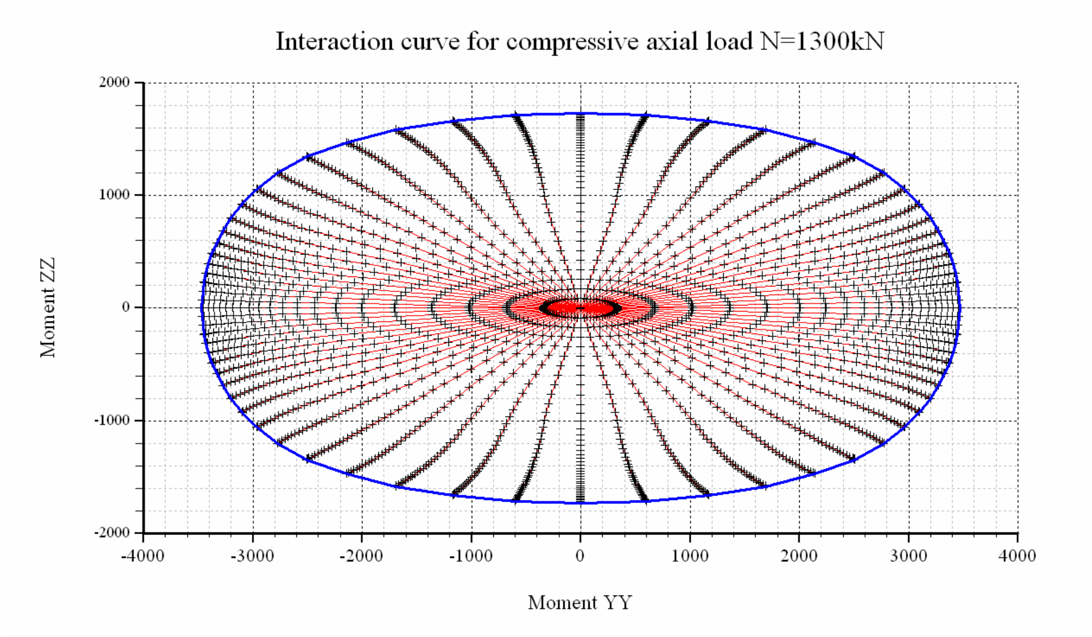


Figure 11. Interaction curve for compressive axial load N=1300kN and restriction at midpoint.

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