Degrading Bouc-Wen Models compatible with Plasticity Postulates

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Abstract: *The Sivaselvan-Reinhorn model which is based on Bouc-Wen phenomenological model of hysteresis is explored from an engineering and mathematical perspective. Remarks and modifications of the original model are provided, concerning the hysteretic behavior of systems exhibiting stiffness degradation, strength deterioration and pinching. In addition, analytical solutions for the hysteretic response of the Sivaselvan-Reinhorn model are derived using Gauss' hypergeometric function on the moment – curvature relations. Sivaselvan-Reinhorn model exhibits displacement drift, force relaxation and nonclosure of hysteretic loops when subjected to small amplitude reversals. This nonphysical behavior is eliminated with the introduction of a stiffening parameter in the hysteretic differential equations. Numerical results are provided that demonstrate the significance of the proposed modifications, particularly for seismic excitations.*

1. INTRODUCTION

The Sivaselvan-Reinhorn model is a smooth hysteretic model of Bouc-Wen type describing systems with stiffness degradation, strength deterioration, pinching and gap-closing behavior. It unifies different independent deterioration rules and finds many practical applications. The Sivaselvan-Reinhorn model has been employed successfully to steel frames [1], reinforced concrete beam-column connections [2], soil-pile interaction systems [3], fiber reinforced composites [4] etc.

Apart from its wide applicability, the model displays certain deficiencies both mathematical and physical. In case of stiffness degradation the system's response is independent of the loading history and thus does not account for cumulative deterioration, while in strength degradation and pinching the mathematical relations provided contain inconsistencies. Moreover, the model as all Bouc-Wen type models, exhibits displacement drift, force relaxation and non closure of hysteretic loops when subjected to short unloading–reloading paths, locally violating Drucker's [5], or Illiushin's [6] postulates of plasticity. In this work, modifications are proposed that treat these deficiencies and a stiffening term is presented, that deals with the system's nonphysical behavior in case of small amplitude reversals.

2. HYSTERETIC BEHAVIOR

2.1 No degrading Model

According to Sivaselvan-Reinhorn model [7], the hysteretic behavior with no degradation is modeled as two parallel springs; one linear elastic and one elastoplastic spring, changing stiffness upon yielding. The combined stiffness is given as:

$$K = K_{postyield} + K_{hysteretic} \tag{1}$$

where the post-yielding stiffness of the linear elastic spring and the stiffness of the hysteretic spring are expressed as:

$$\mathrm{K}_{postyield} = \alpha K_0, \qquad K_{hysteretic} = (1-\alpha) K_0 \left(1 - \left| \frac{M^*}{M_y^*} \right|^N \left[\eta_1 \operatorname{sgn}\left(M^* \dot{\varphi} \right) + \eta_2 \right] \right) \tag{2}$$

where K_0 is the total initial stiffness; α the post-yielding to initial stiffness ratio, N the power controlling the transition from elastic to inelastic range; η_1 and η_2 the parameters controlling the shape of the unloading curve;

M* the portion of the total applied moment shared by the hysteretic spring; $M_y^{*}=(1\text{-}a)M_y$, hysteretic spring's yield moment.

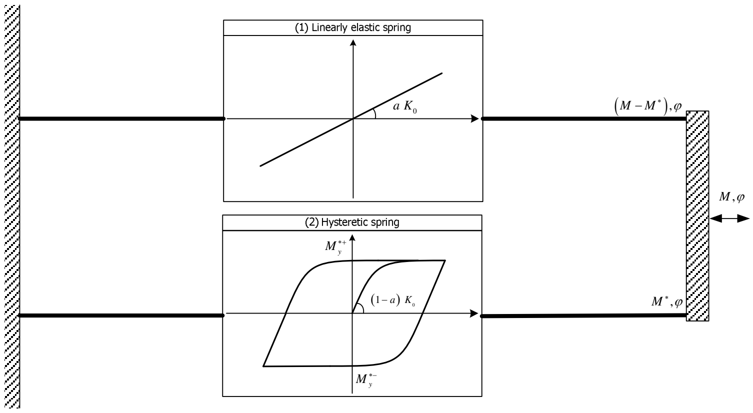

Fig.1 Two-Spring parallel model representation

2.2 Stiffness Degradation

Stiffness degradation is modeled using the pivot rule, introduced by Park et al.[8] The pivot point is on the initial elastic branch at a distance of $\alpha_1 M_y$ on the opposite side, where α_1 is the stiffness degradation parameter.

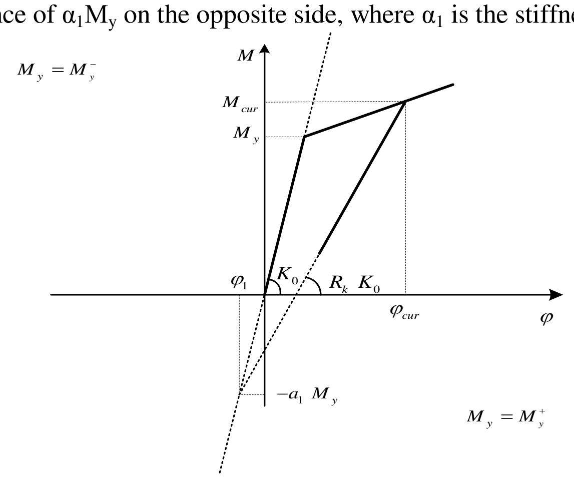

Fig. 2 Park's pivot rule

The stiffness degradation is accounted for through a stiffness degradation factor R_k, introduced in the nondegrading stiffness of the hysteretic spring as:

$$K_{hysteretic} = (R_k - \alpha) K_0 \left(1 - \left| \frac{M^*}{M_y^{\ *}} \right|^N \left[\eta_1 \operatorname{sgn}\left(M^* \dot{\varphi}\right) + \eta_2 \right] \right) \tag{3}$$

The stiffness degradation factor R_k is defined as:

$$R_k = \frac{M_c + \alpha_1 \mathrm{M}_y}{K_0 \varphi_c + \alpha_1 M_y} \tag{4}$$

where M_c is the current value of moment; φ_c the current curvature; K_0 the initial elastic stiffness, α_1 the stiffness degradation parameter and $M_y=M_y^+$ if the current point (M_c, φ_c) is on the right side of the initial elastic branch while $M_y=M_y^-$ if the current point is on the left side of that branch following $\dot{\varphi}>0$ and $\dot{\varphi}<0$ respectively.

From Eq.(4), it turns out that the degradation factor R_k depends only on the current values of moment and curvature[10] and thus does not depend on the history of loading. As such, it should be applied only in case of increasing loading amplitudes, as in decreasing ones the system is stiffened which is not usually the case.

Instead a value R_k^{trial} is evaluated [13] using Eq. (4) and the minimum value of R_k is stored in $R_k^{\min}$:

$$R_k^{\ \min} = \min\left[R_k^{\ \min}, R_k^{\ trial} \right] \tag{5}$$

The degradation parameter R_k is interpolated between $R_k^{\min}$ and R_k^{trial}:

$$R_k = R_k^{\,trial} + (1 - a_2) * (R_k^{\,\min} - R_k^{\,trial}) \tag{6}$$

From (6) , if α_2=1 then R_k=R_k^{trial} and the system is identical to the original one, whereas if α_2=0 the system's stiffness diminishes continuously, since $R_k = R_k^{\,\min}$. The response of a sdof system with unit mass subjected to Erzikan –EW and Kobe- Takatori seismic excitations is presented in Fig. 2 exhibiting significantly different behavior in stiffness degradation with respect to the degradation factor of Eq. (4) and Eq. (6) respectively.

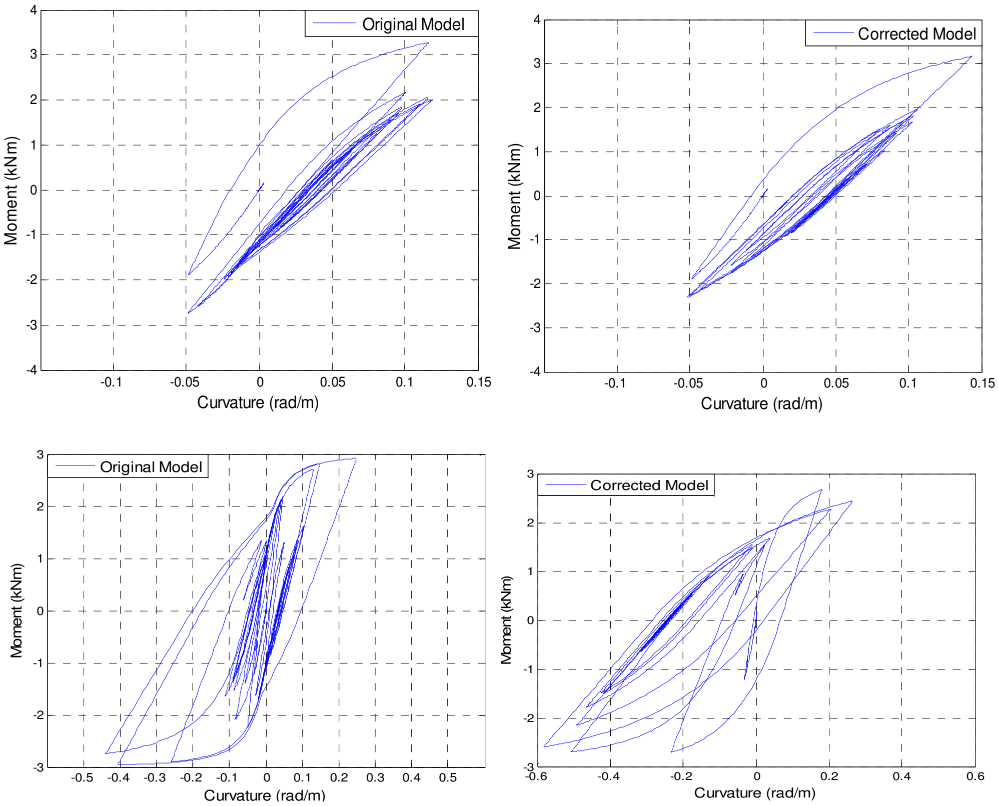

Fig. 2 Hysteresis Loops with different stiffness degradation

It becomes evident that the response in moment-curvature terms differs considerably. Moreover, the parameter α_2 can be directly estimated depending on a system's properties. Introducing parameter a_2 both stiffness degrading and stiffness recovering cases can be successfully modeled.

2.3 Strength Deterioration

Strength deterioration is modeled by reducing the system's capacity and occurs due to dissipation of energy and maximum deformation observed. The strength deterioration rule in Sivaselvan-Reinhorn model [7] reads;

$$M_y^{+/-} = M_{y0}^{+/-}\left[1-\left(\frac{\phi_{\max}^{+/-}}{\phi_u^{+/-}}\right)^{1/\beta_1}\right]\left[1-\frac{\beta_2}{1-\beta_2}\frac{\mathrm{H}}{\mathrm{H}_{ult}}\right] \tag{7}$$

where $M_y^{+/-}$ is the positive or negative yield moment; $M_{y0}^{+/-}$ is the initial positive or negative yield moment; $\varphi_{max}^{+/-}$ is the maximum positive or negative curvatures; $\varphi_u^{+/-}$ is the positive or negative ultimate moments and H is the hysteretic energy dissipated, while H_{ult} is the hysteretic energy dissipated when loaded monotonically to the ultimate curvature without any degradation; β_1 is a ductility-based strength deterioration parameter; and β_2 is an energy-based strength degradation parameter.

Moreover, the hysteretic energy in rate form is given by Sivaselvan-Reinhorn [7] as:

$$\dot{H} = M\left(\dot{\phi} - \frac{\dot{\mathrm{M}}}{R_k K_0}\right) = M\dot{\phi}\left[1 - \frac{\left(K_{postyield} + R_k K_{hysteretic}\right)}{R_k K_0}\right] \tag{8}$$

Eq. (7) is proved mathematically inconsistent [13] and Mostaghel's [9] strength deterioration rule is adopted, which reads:

$$M_y^{+/-} = M_{y0}^{+/-}\left(\frac{1}{1+\beta_{m1}H}\right) \tag{9}$$

$$M_y^{+/-} = M_{y0}^{+/-} e^{-\beta_{m2}H} \tag{10}$$

where $M_{y0}^{+/-}$ is the initial value of yield moment; β_{m1}, β_{m2} the strength deterioration parameters; H the dissipated energy. Differentiating Eq.(9) with respect to time we obtain;

$$\frac{d}{dt}M_y^{+/-} = M_{y0}^{+/-}\left(\frac{1}{\left(1+\beta_{m1}H\right)^2}\right)\dot{H} \tag{11}$$

where $\dot{H}$ is the rate of the dissipated energy given in Eq. (8).

2.4 Pinching

Pinching usually results from crack closure, bolt slip [7] etc. It is conceived as an additional spring connected in series to the hysteretic spring of the parallel model as presented in Fig. 3. According to Sivaselvan-Reinhorn model [7] the stiffness of the slip-lock spring is given as:

$$\mathrm{K}_{sliplock} = \left\{\sqrt{\frac{2}{\pi}}\frac{s}{M_\sigma^*}\exp\left[-\frac{1}{2}\left(\frac{M^*-\bar{M}^*}{M_\sigma^*}\right)^2\right]\right\}^{-1} \tag{12}$$

where s is the slip length, equal to $R_s\left(\phi_{\max}^+ - \phi_{\max}^-\right)$; $M_\sigma^* = \sigma M_y^*$ is a measure of the moment range over which slip occurs; $\bar{M}^* = \lambda M_y^*$ is a mean moment on either side about which slip occurs and R_s, σ, λ are parameters of the model. Moreover, the stiffness of the slip-lock element should be distributed following a Gaussian or any other distribution such as:

$$\int_{-\infty}^{\infty}\frac{1}{K_{sliplock}}dM = s \tag{13}$$

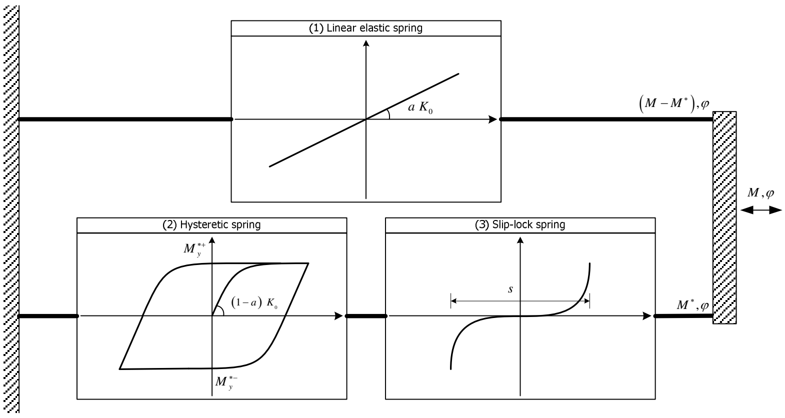

Fig. 3 Serial-parallel model for pinching

The system's combined stiffness is expressed as:

$$K = K_{postyield} + \frac{K_{hysteretic}\,K_{slip-lock}}{K_{hysteretic}+K_{slip-lock}} \tag{14}$$

Wang & Foliente [10] showed that Eq. (12) results into mathematical inconsistencies and they proposed the following expression:

$$K_{sliplock} = \left\{ \sqrt{\frac{2}{\pi}} \frac{s}{\left|M_{\sigma}^{*}\right|} \exp\left[-\frac{1}{2} \left(\frac{M^{*} - \left|\overline{M}^{*}\right| \operatorname{sgn}(\dot{\varphi})}{\left|M_{\sigma}^{*}\right|} \right)^{2} \right] \right\}^{-1} \tag{15}$$

This still does not conform to the standard form of the Gaussian distribution and is substituted herein by the following expression [13]:

$$K_{sliplock} = \left\{ \sqrt{\frac{1}{2\pi}} \frac{s}{\left|M_{\sigma}^{*}\right|} \exp\left[-\frac{1}{2} \left(\frac{M^{*} - \left|\overline{M}^{*}\right| \operatorname{sgn}(\dot{\varphi})}{\left|M_{\sigma}^{*}\right|} \right)^{2} \right] \right\}^{-1} \tag{16}$$

Using Eq.(16) instead of Eq.(15), it is evident that a sdof system's response is considerably different as shown in Fig. 4 for Kobe-Takatori excitation.

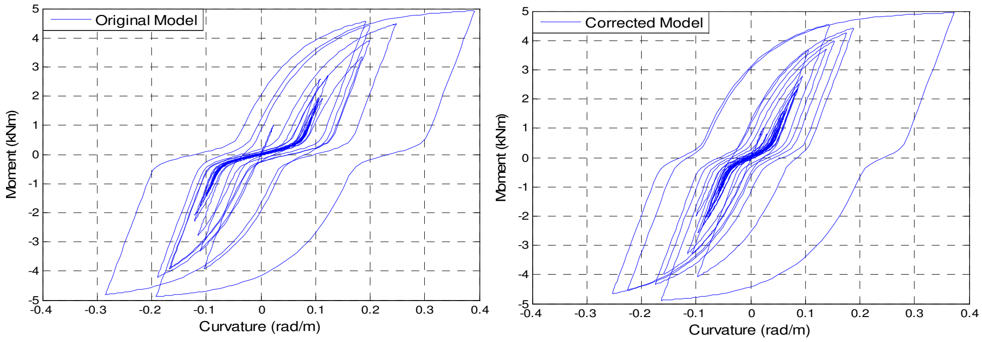

Fig. 4 Hysteresis Loops with different modeling for pinching (Kobe-Takatori excitation)

The overall response of the proposed model based on Eq. (16) is compatible with the mathematics of Gauss distribution and leads to consistent results.

3. ANALYTICAL RESPONSE

Based on previous work of Charalampakis and Koumousis [11], analytical expressions for the hysteretic response of Sivaselvan-Reinhorn model can be derived. As shown in Fig. 5, the behavior of the model can be partitioned into four segments, depending on the sign of M^{*} / M_{y}^{*} and $\dot{\varphi}$. Points A and C signify sign reversal of velocity $\dot{\varphi}$, whereas points B and D signify sign reversal of hysteretic moment M_{y}^{*}. Since the contribution of the elastic spring is trivial, the analysis presented herein focuses in the response of the hysteretic spring only.

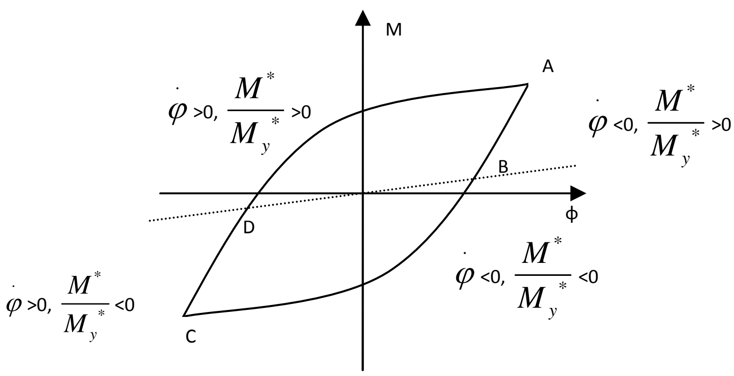

Fig. 5 Response of Sivaselvan –Reinhorn model under cyclic excitation

The yielding moment in rate form is:

$$\dot{M}^* = (1-\alpha)\mathrm{K}_0 \left(1 - \left| \frac{M^*}{M_y^*} \right|^N \left[\eta_1 \operatorname{sgn}\left(M^* \dot{\varphi} \right) + \eta_2 \right] \right) \dot{\varphi} \tag{17}$$

Eliminating time, Eq. (17) can be expressed as:

$$\frac{dM^*}{d\varphi} = (1-\alpha)\mathrm{K}_0 \left(1 - \left| \frac{M^*}{M_y^*} \right|^N \left[\eta_1 \operatorname{sgn}\left(M^* \dot{\varphi} \right) + \eta_2 \right] \right) \tag{18}$$

The indefinite integral of Eq.(18) can be expressed analytically in terms of Gauss' hypergeometric function ${}_2F_1(a,b,c;w)$. Setting M^* / M_y^* equal to z, then Eq.(17) can be expressed as:

$$d\varphi = \frac{M_y^* dz}{(1-\alpha)\mathrm{K}_0 \left(1 - |z|^N q \right)} \tag{19}$$

as it is known that

$$z = \frac{M^*}{M_y^*} \Rightarrow M^* = zM_y^* \Rightarrow dM^* = M_y^* dz \tag{20}$$

Note that Eq.(20) is defined only if the system's yield moment is fixed during excitation. Accounting for initial conditions, Eq.(18) can be written in the form[14]:

$$\varphi - \varphi_0 = \left. \frac{M^*_y z_2 F_1\left(1, \frac{1}{N}; 1 + \frac{1}{N}; q|z|^N \right)}{(1-a)K_0} \right|_{z_0}^{z} \tag{21}$$

where $q = \eta_1 \operatorname{sgn}\left(M^* \dot{\varphi} \right) + \eta_2$ and φ_0 is curvature's initial value. In addition, setting $\varphi_y^* = \dfrac{M_y^*}{(1-a)K_0}$ in Eq.(21) the following expression is obtained:

$$\frac{\varphi - \varphi_0}{\varphi_y} = z_2 F_1\left(1, \frac{1}{N}, 1 + \frac{1}{N}; q|z|^N \right) \Bigg|_{z_0}^{z} \tag{22}$$

Eq.(22) can be solved analytically for z for specific values of the exponential parameter N. For N=1 and N=2 z is given respectively as:

$$z = \frac{\operatorname{sgn}(z) + \left(qz_o - \operatorname{sgn}(z) \right) e^{-\frac{\operatorname{sgn}(z) q (\varphi - \varphi_o)}{\varphi_y}}}{q} \tag{23}$$

$$z = \frac{\tanh\left(\sqrt{q}\,(\varphi - \varphi_o)/\varphi_y + \operatorname{arctan}\left(\sqrt{q} z_o \right) \right)}{\sqrt{q}} \tag{24}$$

where $\tanh(\cdot)$ and $\operatorname{arctan h}(\cdot)$ are the normal and inverse hyperbolic tangent, respectively. In Eq.(24), $\sqrt{q}$ might be complex but the result is real. For arbitrary values of N, Eq.(21) must be solved numerically.

If $\eta_1 = \eta_2 = 1/2$ then the unloading paths are straight lines and z parameter is given by

$$z = \frac{(\varphi - \varphi_o)}{\varphi_y} + z_0 \tag{25}$$

4. CONFORMITY WITH PLASTICITY POSTULATES

4.1 Violation of Plasticity Postulates

Sivaselvan-Reinhorn model exhibits force relaxation, displacement drift and nonclosure of hysteretic loops when subjected to short unloading-reloading paths. This is attributed to the fact that it predicts reduced loading stiffness as compared to the unloading stiffness at the same point, as shown in Fig. 6. As a consequence, negative total work is produced, as expressed in the shaded area in Fig. 6. This violates the Drucker and Illiushin postulates that demand nonnegative work in a closed stress or strain loop respectively and must hold for most isotropic materials, but do not apply necessarily for granular materials.

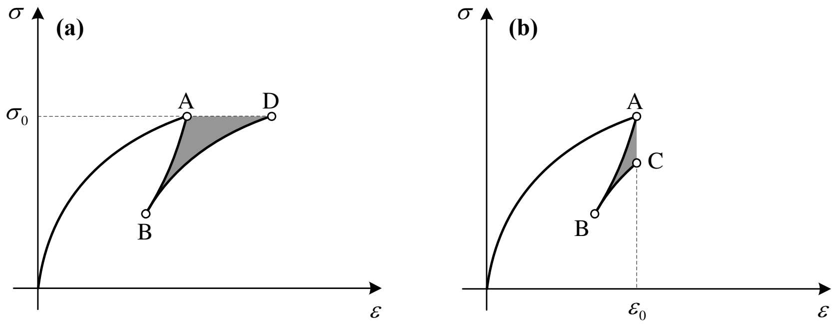

Fig. 6 (a) Displacement drift and (b) force relaxation

4.2 Stiffening Factor

These deficiencies have also been reported for the Bouc-Wen model [12]. To eliminate the aforementioned unrealistic behavior, a modification is proposed based on a mechanism controlling the system's stiffness as follows:

$$\dot{M}^*(t)=(1-a)K_0\left[1-\left|\frac{M^*}{M_y^*}\right|^n\left(n_1+\left(\mathrm{sgn}\left(\dot{\varphi}(t)\mathrm{M}^*(t)\right)\underline{-2H\left(\dot{\varphi}(t)\frac{\mathrm{M}^*(t)}{\mathrm{M}_y^*(t)}\right)R_{s\,\mathrm{mod}}\left(\varphi,\frac{\mathrm{M}^*(t)}{\mathrm{M}_y^*(t)}\right)}\right)n_2\right)\right]\dot{\varphi}(t) \tag{26}$$

where $R_{s\,\mathrm{mod}}$ is the introduced stiffening factor and H is the Heaviside function.

The Heaviside function equals zero in the unloading branches and thus the modified model is identical to the initial one. For $R_{smod}=0$ Eq.(26) reduces to that of the initial model. For $R_{smod}=1$ the loading path's stiffness becomes equal to that of the unloading at the same point.

The stiffening factor R_{smod} is defined as illustrated in Fig. 7. In illustration, we set the reversal point P^+, (φ_p^+ ,M_p^+)in the upper half plane of the M-φ space. When unloading occurs, the system's current position is represented by point A, (φ, M) with $0\leq M< M_p^+$. Point C is the corresponding point of the unloading path.

The unloading path from the reversal point P^+ is known in analytical form and thus, using Eq. (22) point C's curvature can be evaluated for $\eta_1\neq\eta_2$ and $\eta_1=\eta_2$ respectively as:

$$\frac{\varphi_c-\varphi_p^+}{\varphi_y}=\frac{M}{M_y^+}\;{}_2F_1\left(1,\frac{1}{N},1+\frac{1}{N};q\left|\frac{M}{M_y^+}\right|^N\right)\Bigg|_{z_p^+}^{z}\quad\text{, for } \eta_1\neq\eta_2 \tag{27}$$

$$\frac{M_c}{M_y^+}=\frac{\left(\varphi-\varphi_p^+\right)}{\varphi_y}+\frac{M_p^+}{M_y^+},\ \text{for } \eta_1=\eta_2 \tag{28}$$

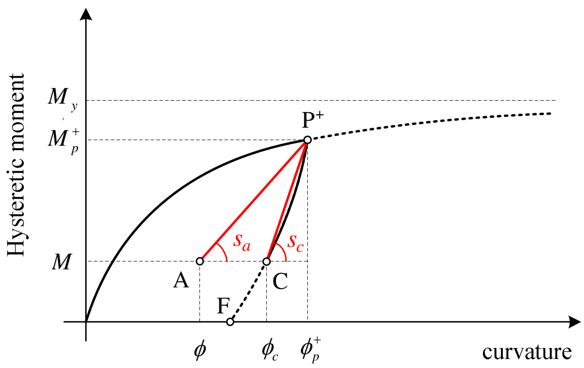

Fig. 7 Formulation of the stiffening factor R_{smod}

Referring to the slope of AP$^+$ line denoted as s_a and s_c the one of line CP$^+$, the formulation of R_{smod} can be based on the ratio of the two slopes aiming at controlling the stiffness in the M-φ space:

$$\frac{s_a}{s_c} = \frac{\dfrac{1}{M_y^+}\left(\dfrac{M_p^+ - M}{\varphi_p^+ - \varphi}\right)}{\dfrac{1}{M_y^+}\left(\dfrac{M_p^+ - M}{\varphi_p^+ - \varphi_c}\right)} = \frac{\varphi_p^+ - \varphi_c}{\varphi_p^+ - \varphi} \tag{29}$$

Therefore, the proposed expression of R_{smod} is as follows:

$$R_{s\,\mathrm{mod}} = H\left(\frac{M_p^+}{M_y^+} - \frac{M}{M_y^+}\right) H\left(\varphi_c - \varphi\right)\left(\frac{\varphi_p^+ - \varphi_c}{\varphi_p^+ - \varphi}\right)^p \tag{30}$$

If the curvature becomes greater than that of the corresponding point on the elastic branch or if the current moment is greater than that of the reversal point then R_{smod} becomes equal to zero, due to the Heaviside functions in Eq.(30), and the stiffening effect disappears. As point A approaches point C from the left, factor R_{smod} increases and approaches unity. When points A and C coincide, R_{smod} =1 and loading follows the unloading path exactly.

Parameter p controls the intensity of stiffening to the left of the unloading path. For increased values of p, stiffening is concentrated close to the unloading path diminishing everywhere else. In general, values of p between 1.0 and 2.0 result into realistic hysteretic behavior.

4.3 Selection of Reversal Points

In case of a single reversal point the effect of the modification is straightforward. Nevertheless, in case of random excitation there are multiple reversal points and the critical issue is which reversal point should be used [12].

When a reversal point P$^+$ is established, a symmetric zone is defined in M-φ space where $M \in (M_p^+, -M_p^+)$. Within this zone, P$^+$ is ''active'' (Fig. 8) in the sense that any single unloading–reloading path of the original Sivaselvan-Reinhorn model falls below P$^+$. At the limit, a path for which the current moment varies in the sequence $M_p^+ \rightarrow -M_p^+ \rightarrow M_p^+$ will be guided to P$^+$ exactly. Based on these observations, stiffening is required for excursions within this zone, so that the path of the hysteretic response will be guided either through or over P$^+$. If the current moment somehow falls outside this zone, P$^+$ is not considered active for the remaining process. Based on this formulation, the set of ''active'' reversal points at time t_i is defined as

$$\bar{T}_i^+ = \left\{ t_j \in T_i^+ \;\middle|\; M\left(t_k\right) \in \left(-M\left(t_j\right), M\left(t_j\right)\right),\ \forall t_k \in \left\{t_{j+1}, t_{j+2}, \ldots, t_i\right\} \right\} \subseteq T_i^+ \tag{31}$$

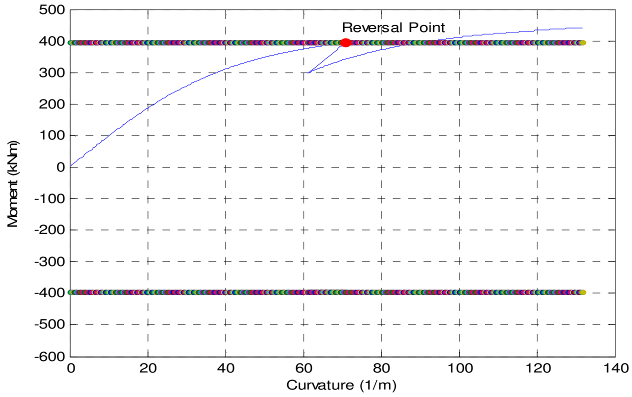

Fig. 8 Active zone of Reversal Point

which contains the time instants that correspond to reversals for which the current moment remains within their respective ''active'' zone up to t_i. At each time instant t_i, the stiffening factors R_{smod} that correspond to all $t_i+ \in \overline{T}_i^+$ are evaluated and the maximum one is used.

For a system with the following characteristics:

α=0.05, M_y=±200 kNm, N=2, K_0=20 kN/m, η_1=η_2=0.5, P=1, the modification is the one presented in Fig. 9

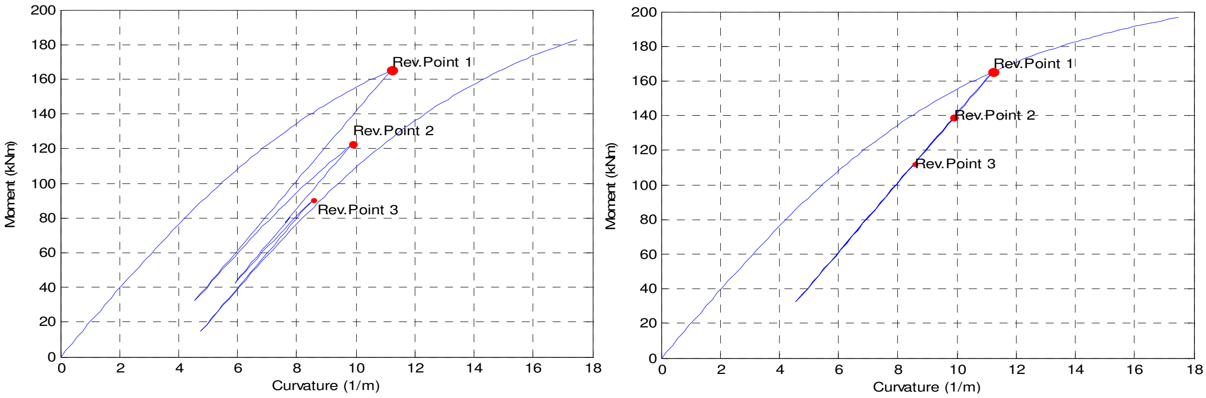

Fig. 9 Application of modified model in case of many reversal points

5. NUMERICAL EXAMPLE

To demonstrate the combined effect of the proposed modifications to the Sivaselvan-Reinhorn model [7], a sdof system with unit mass was subjected to a number of seismic excitations. In Fig. 10, the response of the system to Northridge Tarzana 090 excitation is presented.

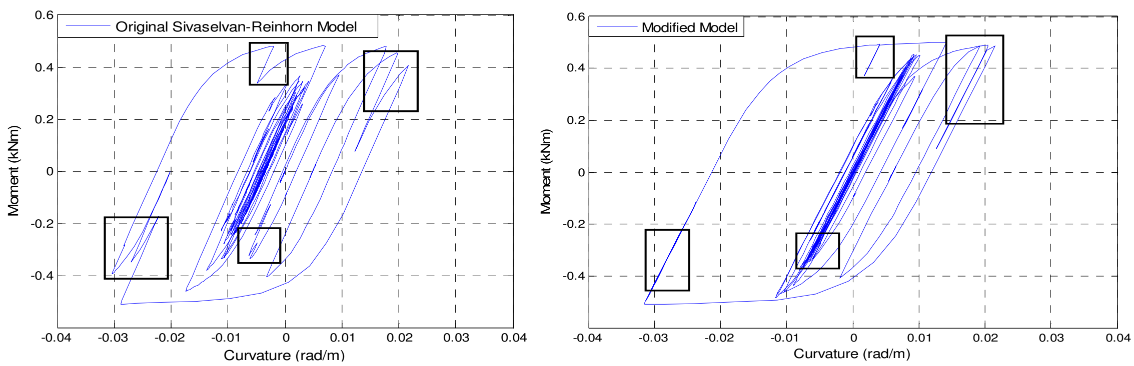

Fig. 10 Initial and Modified Response under the Northridge Tarzana 090 excitation

The critical areas of the response are framed focusing on the effect of modifications. The overall response is affected considerably. Displacement drifts are removed and short reversals non closure is recovered.

Implementation of the proposed modifications towards a consistent degrading hysteretic model of Bouc-Wen type are straight forward and conformity with plasticity postulates can be easily introduced into existing codes that handle Bouc-Wen models by adding the stiffening term.

6. CONCLUSIONS

Bouc-Wen type hysteretic models are phenomenological rate plasticity models based on a number of easily identifiable parameters. They are capable of expressing effectively the inelastic- hysteretic response of a wide range of engineering materials, members and structural component behavior. Degrading models of Bouc-Wen type are more realistic at the cost of identifying some extra parameters. In this work modifications on the stiffness degradation factor of the Sivaselvan-Reinhorn model are proposed, as well as in the expression of the distribution related to pinching while Mostaghel's strength deterioration model is adopted to result into an integrated degrading model. Furthermore, a stiffening term based on the analytical relations is introduced capable of correcting violations of plasticity postulates that raised considerable criticism of Bouc-Wen models in the last decades.

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