COMPDYN 2009
ECCOMAS Thematic Conference on
Computational Methods in Structural Dynamics and Earthquake Engineering
M. Papadrakakis, N.D. Lagaros, M. Fragiadakis (eds.)
Rhodes, Greece, 22–24 June 2009

IMPLEMENTING AN IMPROVED BOUC-WEN MODEL TO ACCOUNT FOR PLASTICITY POSTULATES

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Keywords: Bouc-Wen, Drucker's postulate, Ilyushin's postulate, displacement drift, force relaxation.

Abstract. *The versatile Bouc-Wen model has been used extensively to describe hysteretic phenomena in various fields of engineering. Nevertheless, it is known to exhibit displacement drift, force relaxation and nonclosure of hysteretic loops when subjected to short unloading – reloading paths. Consequently, it locally violates Drucker's or Ilyushin's postulate of plasticity. In this study, an effective modification of the model is implemented which eliminates these problems. A stiffening factor is introduced into the hysteretic differential equation which enables the distinction between virgin loading and reloading. Appropriate reversal points are utilized effectively to guide the entire process. It is shown that the proposed modification fully corrects the nonphysical behavior of the model. It is further demonstrated that the original and modified model may exhibit significantly different response under seismic excitation.*

1 INTRODUCTION

The Bouc-Wen model is a smooth model used to describe hysteretic phenomena. It was introduced by Bouc [1] and extended by Wen [2], who demonstrated its versatility by producing a variety of hysteretic patterns. Although developed independently, it belongs to the class of endochronic models, first introduced by Valanis [3], which use the notion of intrinsic time to describe the inelastic behavior of materials.

The Bouc-Wen model has been employed successfully in many areas of engineering. Nevertheless, it is known that it suffers from nonclosure of hysteretic loops, displacement drift and force relaxation when subjected to short unloading – reloading paths [7]-[10]. The reason for this unrealistic behavior is that the model predicts reduced reloading stiffness as compared to the unloading one, while experimental results show that the two values should be approximately equal [8]. In other words, the Bouc-Wen model does not differentiate between virgin loading and reloading [8]. As a result, it locally violates Drucker's [4] (or Ilyushin's [5]) postulate of plasticity. These postulates are of paramount importance in classical elastoplasticity as they imply the normality rule for the plastic strain rate and the convexity of the yield surface in stress space. Ilyushin's postulate is less restrictive and is applicable to both softening and hardening materials, while resulting in the same consequences as Drucker's [6].

To cope with the violation of these postulates, a modification of the Bouc-Wen model was proposed by Casciati [9]. It involves the introduction of an additional hysteretic term which becomes effective when reloading and gives rise to a plastic displacement in opposite direction with respect to the one produced by the normal hysteretic term. This modification results in the reduction, yet not in the elimination of the violations of plasticity postulates [8], [10].

Notably, these violations can also be reduced by using a large value of the exponential parameter of the model [8], [10]. However, this results in an almost bilinear behavior and, thus, for many engineering problems this approach is deemed as inappropriate.

In this study, the implementation of a simple modification is presented that eliminates the aforementioned unrealistic behavior of the Bouc-Wen model. The modification has been studied in a previous work [18] and focuses directly at the root of the problem, i.e. the reduced reloading stiffness, by inserting a stiffening factor into the hysteretic differential equation. Following a suitable formulation, the modified model incorporates the observation that reloading after partial unloading should follow the unloading path up to the reversal point. Similar remedy was proposed by Riddell and Newmark [11] to correct the nonphysical behavior of Clough's original model [12]. It is shown that the proposed modification eliminates the unrealistic behavior of the Bouc-Wen model with respect to short unloading - reloading paths while leaving its behavior in full hysteretic loops practically unaffected. Guidelines for programming the proposed modification are also presented. Finally, it is shown that, when compared to the original model, the modified model may exhibit significantly different response under random excitation.

2 ORIGINAL MODEL FORMULATION

The restoring force $F(t)$ of a single-degree-of-freedom system can be expressed as:

$$F(t) = a\frac{F_y}{u_y}u(t) + (1-a)F_y z(t) \tag{1}$$

where $u(t)$ is the displacement, F_y the yield force, u_y the yield displacement, a the ratio of post-yield to pre-yield (elastic) stiffness and $z(t)$ a dimensionless hysteretic parameter that obeys a single non-linear differential equation with zero initial condition:

$$\dot{z}(t) = \frac{1}{u_y}\left[A - |z(t)|^n \left(\beta + \operatorname{sgn}\left(\dot{u}(t) z(t)\right)\gamma\right)\right]\dot{u}(t) \tag{2}$$

where A, β, γ, n are dimensionless quantities controlling the behavior of the model, $\operatorname{sgn}(\bullet)$ is the signum function and the overdot denotes the derivative with respect to time. Small values of the positive exponential parameter n correspond to smooth transition from elastic to post-elastic branch, whereas for large values of n the transition becomes abrupt, approaching that of the bilinear model. Parameters β, γ control the size and shape of the hysteretic loop. Parameter A was introduced in the original paper, but it became evident that it is redundant [16].

It follows from Eq. (2) that the restoring force $F(t)$ can be analyzed into an elastic and a hysteretic part as follows:

$$F^{el}(t) = a\frac{F_y}{u_y}u(t) \tag{3}$$

$$F^{h}(t) = (1-a)F_y z(t) \tag{4}$$

Thus, the model can be visualized as two springs connected in parallel (Figure 1) where, $k_i=F_y/u_y$ and $k_f=a\, k_i$ are the initial and post-yielding stiffness of the system.

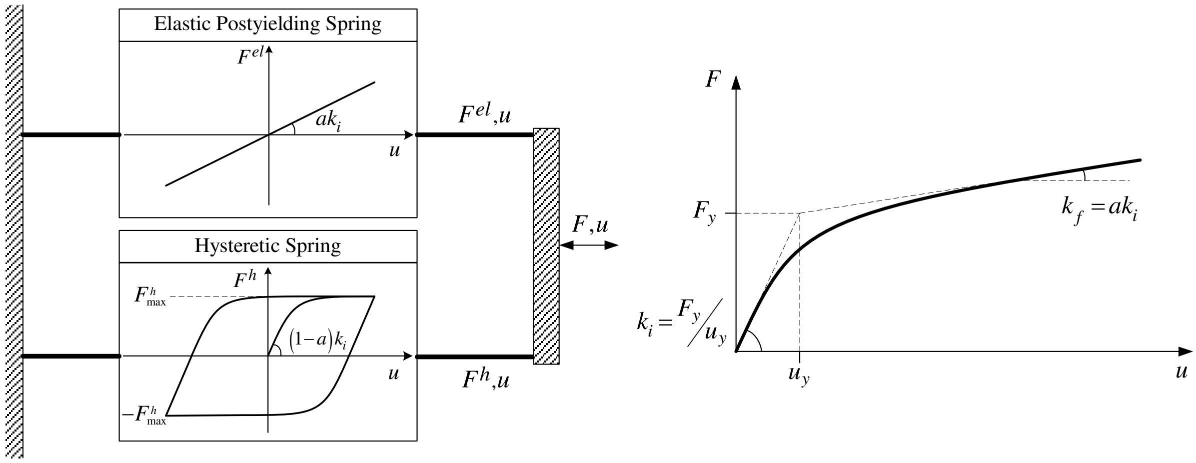

Figure 1: Bouc-Wen model.

Moreover, it has been shown in formal mathematical manner that the parameters of Bouc-Wen model are functionally redundant; there exists a multiplicity of parameter vectors that produce an identical response for a given excitation [16]. Removing this redundancy is best achieved by fixing parameter A to unity [16]. Henceforth, this constraint is assumed to hold.

3 RESPONSE

Recently, analytical expressions for the hysteretic response of Bouc-Wen model were derived [17]. These expressions form the basis of the proposed modification as they provide the full unloading path from a reversal point in analytical form.

The behavior of Bouc-Wen model can be distinguished into four cases depending on the sign of $\dot{u}$ and z. In illustration, the response under cyclic excitation is shown in Figure 2, where the dotted line signifies the path of the elastic response. Points A and C signify sign reversal of velocity $\dot{u}$, whereas points B and D signify sign reversal of hysteretic force F^h or, equivalently, of hysteretic parameter z.

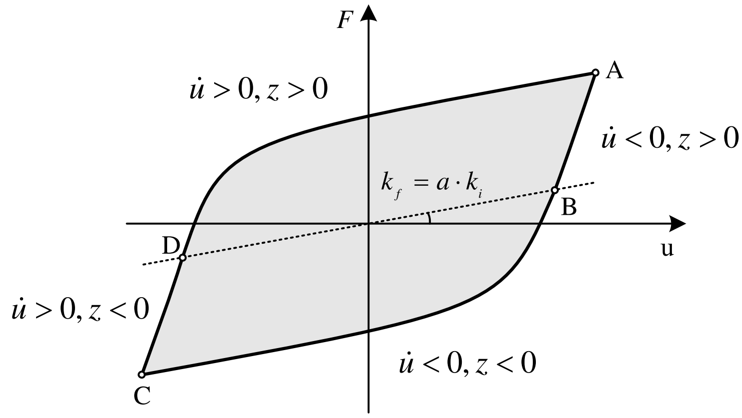

Figure 2: Response of Bouc-Wen model under cyclic excitation.

It was shown that the displacement u is associated with the hysteretic parameter z in terms of Gauss' hypergeometric function ${}_2F_1(a,b,c;w)$ [17]. The following relation holds:

$$\frac{u-u_0}{u_y} = z\ {}_2F_1\left(1,\frac{1}{n},1+\frac{1}{n};q|z|^n\right)\Bigg|_{z_0}^{z} \tag{5}$$

where, $q=\beta+\text{sgn}(\dot{u}z)\gamma$ and u_0, z_0 are the initial values of the displacement and hysteretic parameter, respectively. Eq. (5) can be solved for z analytically for specific values of n, i.e. $n=1$ or $n=2$ [17]. In any case, numerical solution of Eq. (5) is very efficient using bisection-type algorithms [17]. Special attention must be paid with respect to the values of q and sgn(z) per segment (Table 1).

In the special case of β=γ, the unloading branches are straight lines and direct integration of Eq. (2) yields:

$$z = \frac{(u-u_0)}{u_y} + z_0 \tag{6}$$

Eq. (6) is independent of n. The loading branches are covered by Eq. (5).

Segment	q	sgn(z)
AB	β-γ	+1
BC	β+γ	-1
CD	β-γ	-1
DA	β+γ	+1

Table 1: Values of q and sgn(z) per segment.

4 MODIFIED MODEL

The main problem is that the model predicts reduced loading stiffness as compared to the unloading one at the same point. Thus, a mechanism for controlling the stiffness between these two extreme values is needed. To this purpose, Eq. (2) is modified as follows [18]:

$$\dot{z} = \frac{1}{u_y}\left[A - |z|^n\left(\beta + \left(\operatorname{sgn}(\dot{u}\, z) \underline{-2H(\dot{u}\, z) R_s(u,z)}\right)\gamma\right)\right]\dot{u} \tag{7}$$

where the underlined expression is the modification, $R_s(u,z)\in[0,1]$ is a stiffening factor and $H(\bullet)$ is the Heaviside function, due to which the unloading branches of the modified model remain identical to those of the original one. When loading or reloading, factor $R_s(u,z)$ controls the transition between loading (reduced) stiffness and unloading (increased) stiffness. For $R_s = 0$, Eq. (7) reduces to Eq. (2) and the proposed modified model is identical to the original one. For $R_s = 1$, the loading stiffness becomes equal to that of unloading at the same point.

By virtue of Eqs. (5) and (6), the full unloading path from a reversal point is known *a priori* in analytical form. For $\beta \neq \gamma$ this path is curved, whereas for $\beta = \gamma$ it is a straight line. Imposing that $R_s = 1$ along this path has the desired effect that partial unloading followed by reloading will guide the hysteretic response exactly on the unloading path up to the reversal point. Upon there, factor R_s should revert to zero to allow for further loading with normal (reduced) stiffness. Finally, factor R_s should diminish in regions away of the unloading path so that normal behavior of Bouc-Wen model remains unaffected.

Based on these observations, a suitable expression of $R_s(u,z)$ is determined. In illustration, we assume that $P^+(u_p^+, z_p^+)$ is a reversal point in the upper half-plane of the $u-z$ space ($z_p^+ > 0$). Symmetric formulation with respect to the origin of the reference axes is assumed for the lower half-plane. During reloading, it is assumed that the current state is represented by point $A(u,z)$ with $0 \le z < z_p^+$ (Figure 3). Point $C(u_c, z)$ is the corresponding point of the unloading path. By employing Eqs. (5) and (6), u_c is given by Eqs. (8) and (9) for $\gamma \neq \beta$ and $\gamma = \beta$, respectively, as:

$$u_c(z) = u_y\, z\, {}_2F_1\left(1, \frac{1}{n}, 1+\frac{1}{n}; (\beta-\gamma) z^n\right)\Bigg|_{z_p^+}^{z} + u_p^+ \tag{8}$$

$$u_c^*(z) = \left(z - z_p^+\right)u_y + u_p^+ \tag{9}$$

It is noted that β is considered equal to γ in most cases of interest. Thus, the unloading path is given by Eq. (9) and there is no need to evaluate the hypergeometric function.

A natural way of controlling stiffness in the u - z space is based on the slopes in the same space. We denote s_a the slope of line AP^+, as opposed to the "critical" slope s_c of line CP^+. Referring to Figure 3, it follows that $s_a/s_c = \left(u_p^+ - u_c(z)\right)/\left(u_p^+ - u\right)$. Based on this ratio, a simple expression for the factor R_s was proposed [18] as:

$$R_s(u,z) = H\left(z_p^+ - z\right) H\left(u_c(z) - u\right)\left(\frac{u_p^+ - u_c(z)}{u_p^+ - u}\right)^p \tag{10}$$

where $p \ge 1$ is a constant. As point A approaches point C from the left, factor R_s increases and approaches unity. When points A and C coincide, $R_s = 1$ and loading follows the unloading path exactly. Thus, the unloading path P^+ - F is a "horizon", i.e. it cannot be crossed. When $z > z_p^+$ or $u > u_c$, the stiffening effect disappears due to the Heaviside functions of Eq. (10). Parameter p controls the intensity of stiffening to the left of the unloading path. For increased values of p, stiffening is concentrated close to the unloading path and diminished everywhere else. In general, it was observed that values of p between 1.0 and 2.0 produce realistic hysteretic behavior.

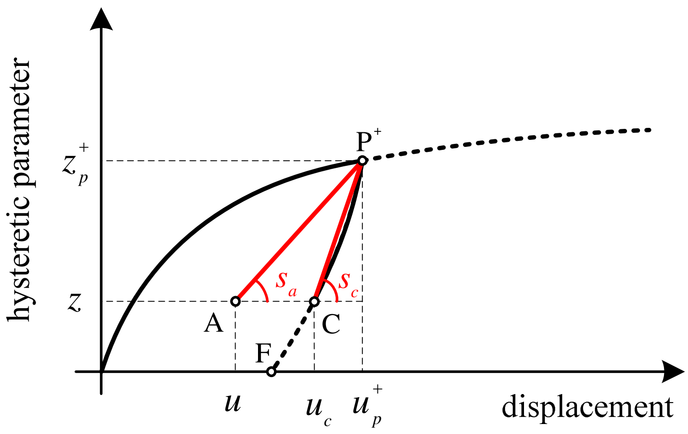

Figure 3: Formulation of stiffening factor R_s.

To demonstrate the effect of the proposed modification, we consider a system with $n = 2$, $\beta = 0.1$, $\gamma = 0.9$ which is subjected to virgin loading. Unloading occurs when $u_p^+ = 1.5u_y$ and $z_p^+ \cong 0.905$. We impose a displacement to the negative direction and then back to the positive direction. Applying the stiffening rule with $p = 2$ has a profound effect on the response of the hysteretic spring. In illustration, Figure 4 shows cases (a) to (d) where loading in the negative direction reaches u_y, $0.5u_y$, 0 and $-1.5u_y$, respectively. It is demonstrated that the differences in the response depend on the intensity of the reversal. In cases (a,b,c), the nonphysical behavior of Bouc-Wen model is corrected, whereas in case (d) the response of the original and modified model are practically identical.

In addition, Figure 5 shows the contour plots of stiffening factor R_s in case of $p = 1$ and $p = 2$. These plots are fully defined upon establishment of reversal point $P^+\left(1.5u_y, 0.905\right)$. The darker a point is, the more intense is the stiffening effect at that point during reloading. The edge of the darkest area is the unloading path, along which $R_s = 1$ irrespectively of p. It is shown that for $p = 2$ stiffening is concentrated close to the unloading path.

5 SELECTION OF REVERSAL POINT

The effectiveness of the proposed modification was demonstrated for the case of a single reversal point. Nevertheless, for a system under random excitation a critical issue arises regarding *which* reversal point should be used.

It has been shown that using either the last observed reversal point or the reversal point that corresponds to maximum displacement leads to a formulation that is ineffective in certain cases [18]. In order to cover all cases, one has to take into account multiple reversal points. Therefore, it is important to investigate the conditions under which a reversal point should be considered “active”.

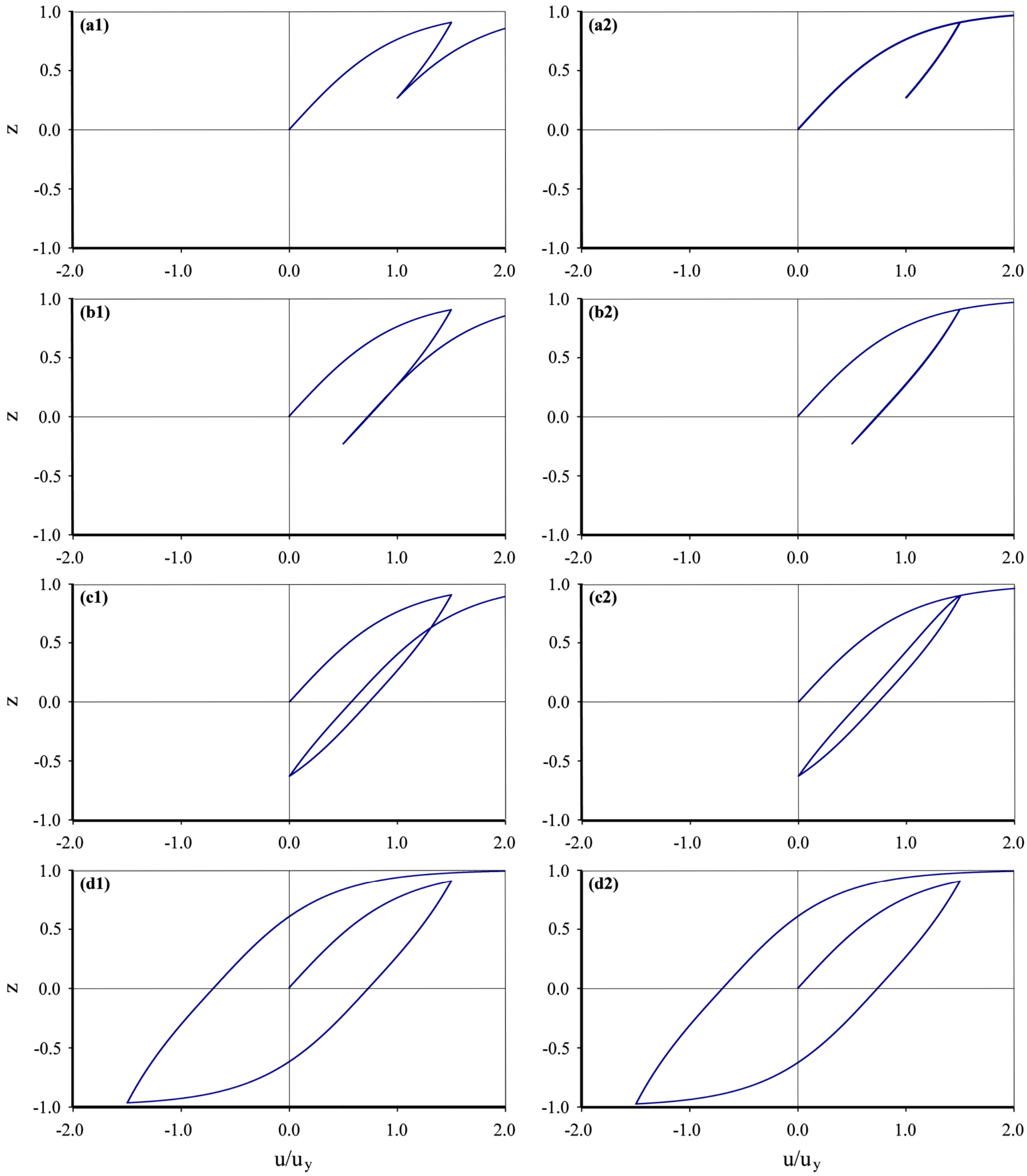

Figure 4: Stiffening effect in u-z space: original model (left), modified model (right) (n=2, β=0.1, γ=0.9, $u_p^+=1.5u_y$, $z_p^+\approx0.905$, p=2).

When a reversal point $P^+\left(u_p^+, z_p^+\right)$ is established, a symmetric zone is defined in $u-z$ space where $z \in \left(-z_p^+, z_p^+\right)$. Within this zone, P^+ is "active" in the sense that any single unloading – reloading path of the original Bouc-Wen model falls below P^+ (Figure 6). At the limit, a path for which the hysteretic parameter varies in the sequence $z_p^+ \to -z_p^+ \to z_p^+$ will be guided to P^+ exactly [18]. Based on these observations, stiffening is required for excursions within this zone, so that the path of the hysteretic response will be guided either through or over P^+. If z somehow falls outside this zone, P^+ is not considered active for the remaining process. In other words, the active reversal points are those for which the hysteretic parameter remains within their respective "active" zone from the time of their establishment up to the present time instant. The stiffening factors that correspond to all active reversal points are evaluated and the maximum one is used.

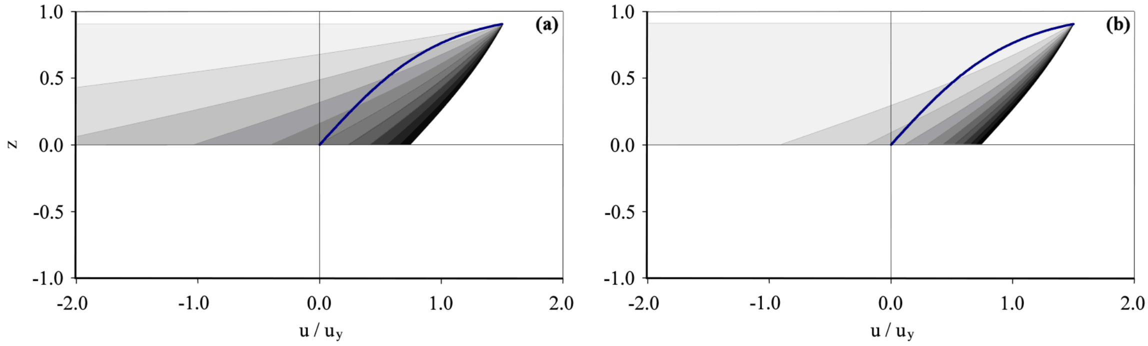

Figure 5: Contour plot of R_s with n=2, β=0.1, γ=0.9, u_p^+=1.5u_y, z_p^+≈0.905 and (a) p=1.0 (b) p=2.0.

To demonstrate the effectiveness of the proposed formulation, we consider a system with the following properties: $\beta = 0.1$, $\gamma = 0.9$, $a = 0.10$, $n = 2.0$, $F_y = 2.86kN$, $u_y = 0.111m$, $m = 13kNs^2/m$ and $p = 2.0$, subjected to Northridge TAR090 [19]. When using the formulation with multiple reversal points, it is observed that all intermediate reversals are correctly ignored (Figure 7). These include the reversals at the end of the event, which cause considerable drift in the original model.

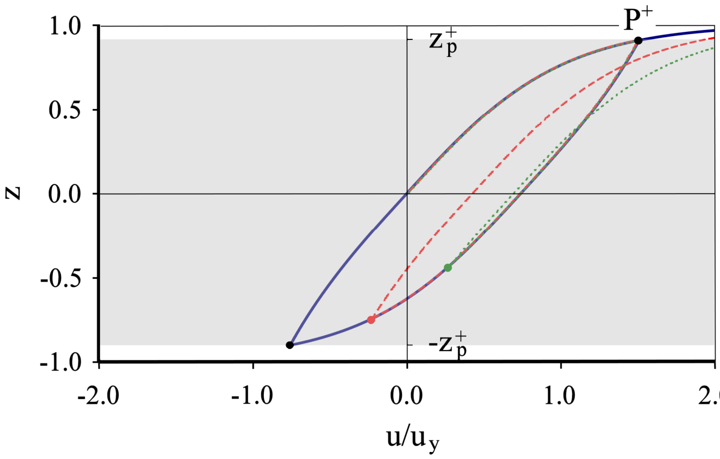

Figure 6: "Active" zone of reversal point P^+ (n=2, β=0.1, γ=0.9, u_p^+=1.5u_y, z_p^+≈0.905).

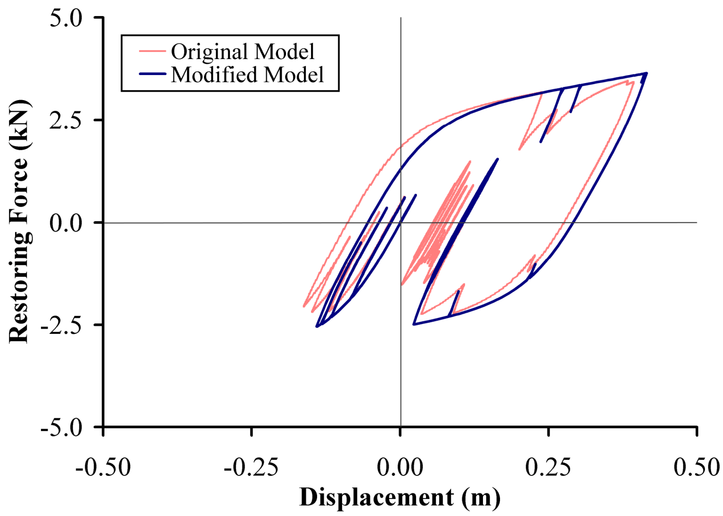

Figure 7: Response under the Northridge TAR090 [19] excitation using multiple reversal points.

6 GUIDELINES FOR PROGRAMMING THE MODIFIED MODEL

Programming of the proposed modification is straightforward and is implemented at each integration step by (a) adding the stiffening term into the differential equation, (b) evaluating and employing the maximum stiffening factor R_s that corresponds to "active" reversal points using relation (10) and (c) updating the set of "active" reversal points. The latter is accomplished effectively by adding into the set the new reversal points and removing existing ones that have become "inactive".

In order to evaluate the response in a force-controlled experiment, the original single-degree-of-freedom Bouc-Wen model with external viscous damping is cast into state-space as follows:

$$\begin{Bmatrix} x_1(t) = u(t) \\ x_2(t) = \dot{u}(t) \\ x_3(t) = z(t) \end{Bmatrix} \tag{11}$$

$$\begin{Bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{Bmatrix} = \begin{Bmatrix} x_2(t) \\ -\dfrac{1}{m}\left[c\, x_2(t) + a\dfrac{F_y}{u_y} x_1(t) + (1-a) F_y x_3(t) - f(t) \right] \\ \dfrac{1}{u_y}\left[\left(A - |x_3(t)|^n \left(\gamma \operatorname{sgn}\left(x_2(t)\, x_3(t) \right) + \beta \right) \right) x_2(t) \right] \end{Bmatrix} \tag{12}$$

where, x_1, x_2 and x_3 are auxiliary variables, c is the linear viscous damping coefficient and $f(t)$ is the external excitation. The derivatives for the modified model are given as:

$$\begin{Bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{Bmatrix} = \begin{Bmatrix} x_2(t) \\ -\dfrac{1}{m}\left[c\, x_2(t) + a\dfrac{F_y}{u_y} x_1(t) + (1-a) F_y x_3(t) - f(t) \right] \\ \dfrac{1}{u_y}\left[\left(A - |x_3(t)|^n \left(\gamma \left(\operatorname{sgn}\left(x_2(t)\, x_3(t)\right) \underline{-2H\left(x_2(t)\, x_3(t)\right) R_s} \right) + \beta \right) \right) x_2(t) \right] \end{Bmatrix} \tag{13}$$

The above system can be integrated numerically using a Runge-Kutta 4th-5th order integrator. Updating of active reversal points is performed after the completion of the evaluation of each time step.

7 ELASTIC AND HYSTERETIC BEHAVIOR OF BOUC-WEN MODEL

For real values of the stiffness it can be proved that the elastic behavior is approximated asymptotically from the inelastic regime, whereas the hysteretic behavior is restricted to thin hysteretic loops. Hysteresis in the linear case can be treated using complex valued stiffness as in [21].

To evaluate the inelastic deformation after an arbitrary loading-unloading circle, we need to decompose the response into phases as in Figure 8, where the dotted line signifies the path of the elastic spring.

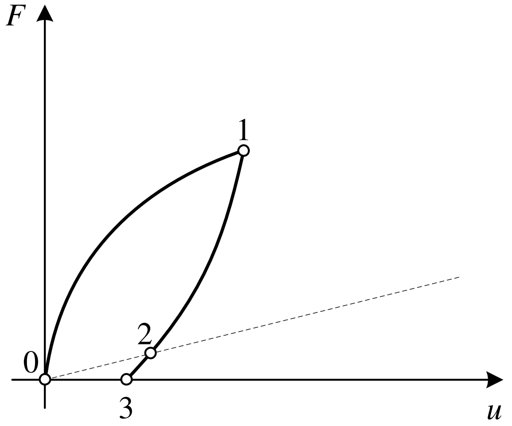

Figure 8: Calculation of inelastic deformation of Bouc-Wen model.

Employing Eq. (5) in the sequence $0 \to 1 \to 2 \to 3$ and noting that $u_0 = 0$, $z_0 = z_2 = 0$, $z_1 > 0$ and $z_3 < 0$, one obtains:

$$\frac{u_1 - 0}{u_y} = z_1\ {}_2F_1\left(1, \frac{1}{n}, 1 + \frac{1}{n}; (\beta + \gamma) z_1^{\ n}\right) - 0 \tag{14}$$

$$\frac{u_2 - u_1}{u_y} = 0 - z_1\ {}_2F_1\left(1, \frac{1}{n}, 1 + \frac{1}{n}; (\beta - \gamma) z_1^{\ n}\right) \tag{15}$$

$$\frac{u_3 - u_2}{u_y} = z_3\ {}_2F_1\left(1, \frac{1}{n}, 1 + \frac{1}{n}; (\beta + \gamma)(-z_3)^n\right) - 0 \tag{16}$$

At final Point 3 the force of the hysteretic spring is opposite to that of the elastic spring, thus:

$$z_3 = -\frac{a\,u_3}{(1-a)u_y} \tag{17}$$

Adding Eqs. (14) to (16) by parts and substituting Eq. (17) yields:

$$\begin{aligned}\frac{u_3}{u_y} + z_1\ {}_2F_1\left(1,\frac{1}{n},1+\frac{1}{n};(\beta-\gamma)z_1^{\,n}\right) &= z_1\ {}_2F_1\left(1,\frac{1}{n},1+\frac{1}{n};(\beta+\gamma)z_1^{\,n}\right) - \\ \frac{a\,u_3}{(1-a)u_y}\ {}_2F_1&\left(1,\frac{1}{n},1+\frac{1}{n};(\beta+\gamma)\left(\frac{a\,u_3}{(1-a)u_y}\right)^n\right)\end{aligned} \tag{18}$$

Eq. (18) can be solved efficiently for the unknown inelastic deformation u_3 using bisection-type numerical methods. For $n=1$, it is solved analytically as:

$$\begin{aligned}u_3 &= \frac{u_y}{a(\beta+\gamma)}\times \\ &\left(1-a-a\ \text{Productlog}\left(\frac{(a-1)e^{\frac{1-a}{a}}\left((\beta+\gamma)z_1-1\right)\left(1-(\beta-\gamma)z_1\right)^{-\frac{\beta+\gamma}{\beta-\gamma}}}{a}\right)\right)\end{aligned} \tag{19}$$

where, $\text{ProductLog}(z)$ is the principal solution for w in $z = we^w$ and $\beta \neq \gamma$ is assumed, so that using the hypergeometric function during unloading $1 \rightarrow 2$ is meaningful. The physical meaning of Eqs. (18) and (19) is that, for an arbitrary loading-unloading circle, there is always a permanent non-zero inelastic deformation which may be small, but nevertheless calculable.

8 COMPARISON OF ORIGINAL AND MODIFIED MODEL

It has been demonstrated that the proposed modification restores the physical consistency of the Bouc-Wen model from a theoretical perspective. What is more important is that the overall response of the modified model may be considerably different from that of the original in case of seismic excitation.

In illustration, we are interested in the peak values of certain time histories for design purposes. We measure the relative error of the peak values as follows:

$$\varepsilon = \frac{\max\left(\overline{y}(t)\right) - \max\left(y(t)\right)}{\max\left(y(t)\right)} \tag{20}$$

where $y(t)$ and $\overline{y}(t)$ are the time histories corresponding to the original and modified model, respectively, and $\max(\cdot)$ denotes the maximum absolute value.

We consider a specific system with the following properties: $\beta = 0.1$, $\gamma = 0.9$, $a = 0.10$, $n = 2.0$, $F_y = 2.86kN$, $u_y = 0.111m$. Regarding the modified model, the formulation with multiple reversal points is employed with $p = 2.0$. The plastic period is controlled by changing the mass of the system. For a selection of 20 strong motion recordings [19] (Table 2), Figure 9 and Figure 10 show the envelope of the relative error in the peak displacement and peak hysteretic energy, respectively. The results have been filtered to include cases for which $\max\left(u\left(t\right)\right)\geq u_y$ or $\max\left(\bar{u}\left(t\right)\right)\geq u_y$. Thus, only the results that involve an appreciable level of hysteretic damping are displayed. It is observed that the peak values of the modified model may be smaller or larger than that of the original one. For the excitations considered herein, the relative error may reach 38% and 24% for the peak displacement and peak hysteretic energy, respectively.

#	Title	PGA (g)	PGV (cm/s)	PGD (cm)
1	ChiChi CHY028 N	0.821	67.0	23.28
2	ChiChi CHY028 W	0.653	72.8	14.68
3	ChiChi TCU084 N	0.417	45.6	21.27
4	ChiChi TCU084 W	1.157	114.7	31.43
5	Kobe Takatori TAK000	0.611	127.1	35.77
6	Kobe Takatori TAK090	0.616	120.7	32.72
7	Northridge Rinaldi RRS228	0.838	166.1	28.78
8	Northridge Rinaldi RRS318	0.472	73.0	19.76
9	Northridge Tarzana TAR090	1.779	113.6	33.22
10	Northridge Tarzana TAR360	0.990	77.6	30.45
11	Kocaeli Duzce DZC180	0.312	58.8	44.11
12	Kocaeli Duzce DZC270	0.358	46.4	17.61
13	Tabas TAB-LN	0.836	97.8	36.92
14	Tabas TAB-TR	0.852	121.4	94.58
15	Imperial Valley I-ELC180	0.313	29.8	13.32
16	Imperial Valley I-ELC270	0.215	30.2	23.91
17	Loma Prieta GPC000	0.563	94.8	41.18
18	Loma Prieta GPC090	0.605	51.0	11.50
19	Erzikan ERZ-NS	0.515	83.9	27.35
20	Erzikan ERZ-EW	0.496	64.3	22.78

Table 2: Strong motion recordings taken from PEER [19].

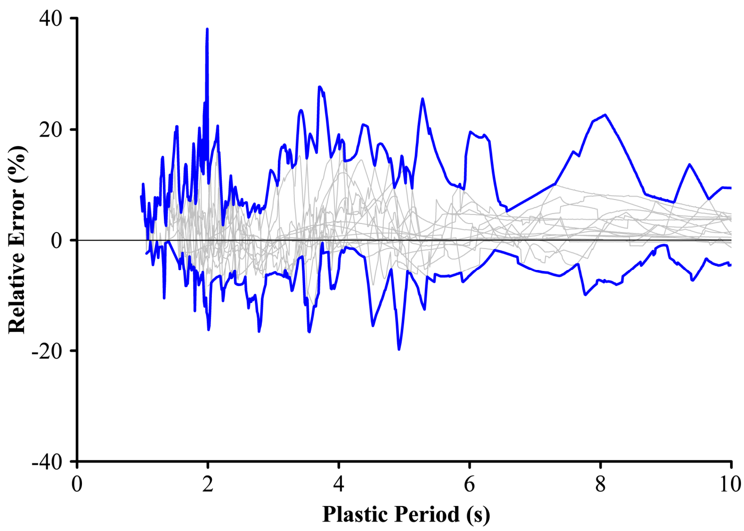

Figure 9: Relative peak displacement error.

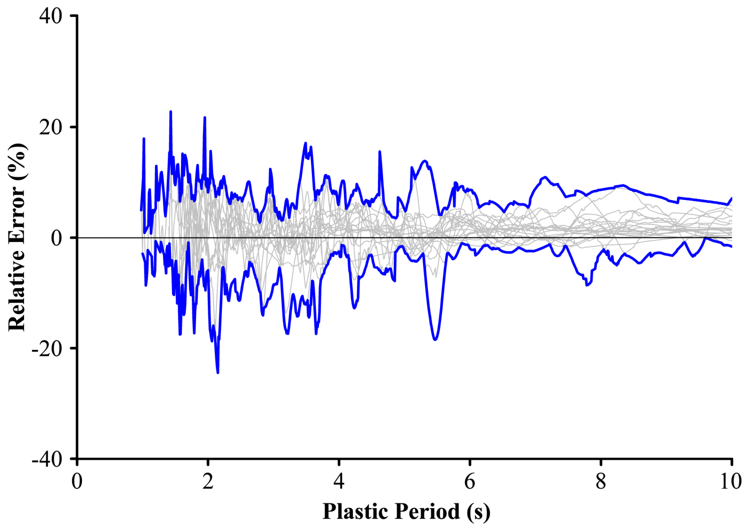

Figure 10: Relative peak hysteretic energy error.

Another example of the corrective effect of the modified model refers to the calculation of the residual displacement. It has been observed that the residual displacement is underestimated by the original Bouc-Wen model due to the oscillation at the end of the event [20]. During this oscillation, the non-physical behaviour of the original model causes significant drift (Figure 11), while the modified model corrects this problem inherently. This is especially important in seismic isolation, where the existing codes demand certain recentering capability of the system and the Bouc-Wen model is used very frequently.

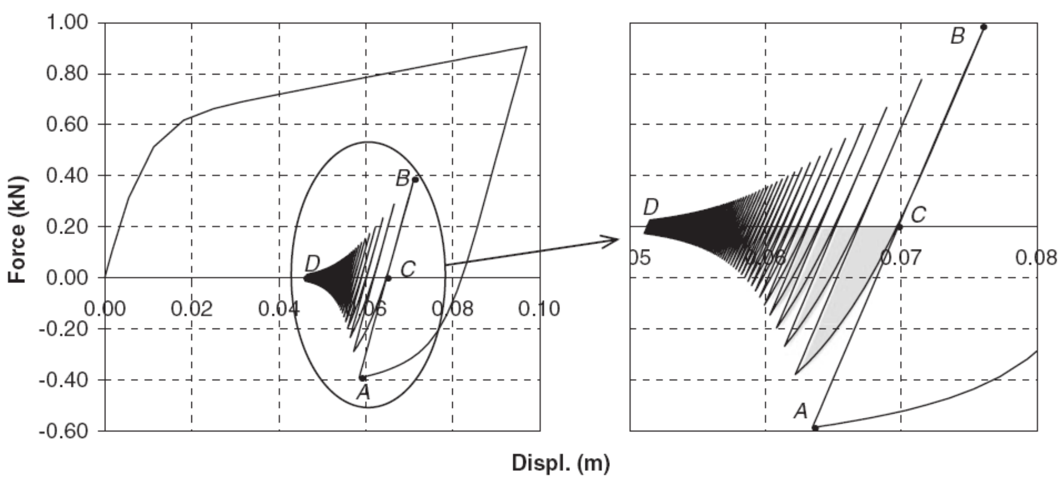

Figure 11: Underestimation of residual displacement due to oscillation at the end of the event [20].

9 CONCLUSIONS

A simple modification of the versatile Bouc-Wen model is proposed which results in the correction of its nonphysical behaviour when subjected to short unloading – reloading paths. This behaviour is manifested as displacement drift, force relaxation and nonclosure of hysteretic loops, which result into violation of Drucker’s or Ilyushin’s postulate. The proposed modification is based on the introduction of a suitable stiffening factor which is inserted directly into the hysteretic differential equation and differentiates virgin loading and reloading, a feature that is absent in the original model. The notion of “active” reversal points is described which controls the entire process effectively. The proposed modifications are explained in detail and their effects are demonstrated and discussed. Moreover, guidelines for an efficient implementation of the proposed modification in computer code are provided. Finally, it is shown that the original and modified model may exhibit significantly different response under seismic excitation.

The proposed modification can be applied to extended Bouc-Wen models that also take into account degradation phenomena, e.g. [15]. This is feasible since the modification focuses in the hysteretic spring only and it is fully formulated within the $u-z$ space.

10 ACKNOWLEDGEMENTS

This work has been funded by the project PENED 2003. The project is co-financed 80% of public expenditure through EC - European Social Fund, 20% of public expenditure through Ministry of Development - General Secretariat of Research and Technology and through private sector, under measure 8.3 of Operational Programme "Competitiveness" in the 3rd Community Support Programme.

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