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ROBUST IDENTIFICATION OF BOUC-WEN HYSTERETIC SYSTEMS BY SAWTOOTH GA AND BOUNDING

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Abstract. *Hysteresis is a term that describes macroscopically many phenomena observed in engineering. The complexity of the actual mechanism behind hysteresis has given rise to the extended use of phenomenological models, such as the Bouc-Wen model. This paper presents a new stochastic identification scheme for Bouc-Wen systems that combines Sawtooth Genetic Algorithm and a Bounding technique that gradually focuses into smaller and better regions of the search space. Numerous studies show that the proposed scheme is very robust and insensitive to noise-corrupted data. Apart from frequency-independent hysteretic characteristics, the method is also able to identify viscous-type damping.*

1 INTRODUCTION

Hysteresis is a phenomenon observed in many fields such as mechanics, magnetism, electricity, materials and elasto-plasticity of solids. In mechanics, systems with components of inherent non-linear nature often exhibit overall hysteretic behavior. Examples include reinforced concrete sections, steel sections, bolted connections, base isolators such as Lead Rubber Bearings (LRB), Friction Pendulum Systems (FPS) etc.

The Bouc-Wen model is a versatile endochronic model that is often used to describe hysteretic phenomena. It was introduced by Bouc [1] in 1967 but it was Wen [2] in 1976 that extended the model and demonstrated its versatility by producing a variety of hysteretic patterns. These were further enhanced by Baber and Noori [3] to incorporate non-symmetric behavior and degradation phenomena, with the unavoidable expense of additional parameters. Since then, researchers have introduced improved Bouc-Wen type models, such as Foliente [4] and Sivaselvan and Reinhorn [5].

In general, identification of Bouc-Wen models poses a challenging problem because of its highly non-linear nature. Researchers have applied a variety of methods, such as Gauss-Newton [6], Modified Gauss-Newton [7], Least squares [8], Simplex [9], Levenberg-Marquardt [9], [10], extended Kalman filters [9], [11], reduced gradient methods [9], Differential Evolution [12], [13], etc.

Various techniques have been used to ameliorate accuracy and convergence problems. Furthermore, in most cases, crucial system parameters such as stiffness and viscous damping are considered known. Considering base isolators, frequency-independent hysteretic damping is dominant and there is no need to employ viscous damping. In other cases, however, viscous-type effects may become significant when the evolution of the phenomenon is fast [10]. Therefore, viscous damping characteristics need to be identified rather than assumed known.

This paper presents a new stochastic identification scheme that addresses the aforementioned issues. The method combines Sawtooth Genetic Algorithm (GA) [14] and a Bounding technique. The latter is a critical improvement upon an earlier method presented by the authors [15]. The results show that the proposed scheme is very robust and insensitive to noise-corrupted data.

2 HYSTERETIC MODEL

2.1 Formulation

Considering a single-degree-of-freedom (SDOF) system, the restoring force *F(t)* can be expressed as:

$$F(t) = a \cdot \frac{F_y}{u_y} \cdot u(t) + (1-a) \cdot F_y \cdot z(t) \tag{1}$$

Where, *u(t)* is the displacement, F_y the yield force, u_y the yield displacement, a the ratio of post-yield to pre-yield (elastic) stiffness and *z(t)* a dimensionless hysteretic parameter that obeys a single non-linear differential equation:

$$\dot{z}(t) = \frac{1}{u_y}\left[A - |z(t)|^n \cdot \left(\gamma \cdot sign\left(\dot{u}(t) \cdot z(t)\right) + \beta\right)\right] \cdot \dot{u}(t) \tag{2}$$

Where, A, β, γ, n are dimensionless quantities controlling the behavior of the model and the overdot denotes the derivative with respect to time. Small values of the positive exponential parameter n correspond to smooth transition from elastic to post-elastic branch, whereas for large values of n the transition becomes abrupt, approaching that of a bilinear model. Parameters β, γ, control the size and shape of the hysteretic loop. Parameter A was introduced in the original paper, but recently it became evident that it is redundant.

It follows from Eq. (1) that the restoring force $F(t)$ can be analyzed into two springs connected in parallel; the first is the linear elastic post-yielding spring and the second is the hysteretic spring (Fig. 1), where, $k_i=F_y/u_y$ is the initial stiffness, $k_f=\alpha k_i$ the post-yielding stiffness and F^* the force of the hysteretic spring.

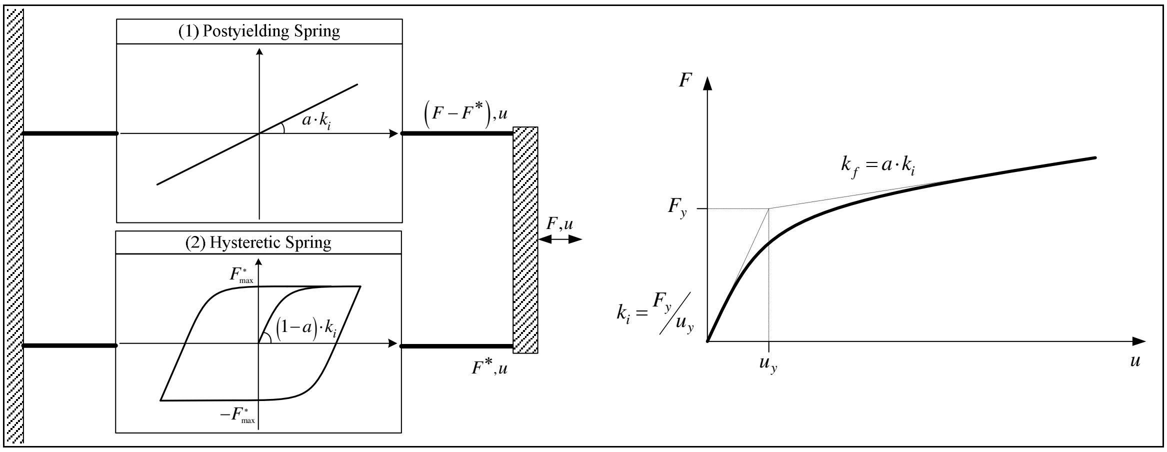

Fig. 1: Bouc-Wen model

Apart from rate-independent hysteresis, viscous-type damping can also be employed. In this case, the equation of motion of a SDOF system becomes:

$$m \cdot \ddot{u}(t) + c \cdot \dot{u}(t) + F(t) = f(t) \tag{3}$$

Where, m is the mass, c the linear viscous damping coefficient and $f(t)$ the excitation force. In displacement-controlled experiments, the time history of the displacement and its derivatives are readily available and the calculation of forces is trivial. In case of force-controlled experiments, by substituting Eq. (1) into (3) one obtains:

$$m \cdot \ddot{u}(t) + c \cdot \dot{u}(t) + a \cdot \frac{F_y}{u_y} \cdot u(t) + (1-a) \cdot F_y \cdot z(t) = f(t) \tag{4}$$

Eqs. (4) and (2) are transformed into state-space form as follows:

$$\begin{Bmatrix} x_1(t) = u(t) \\ x_2(t) = \dot{u}(t) \\ x_3(t) = z(t) \end{Bmatrix} \tag{5}$$

$$\begin{Bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{Bmatrix} = \begin{Bmatrix} x_2(t) \\ -\frac{1}{m} \cdot \left[c \cdot x_2(t) + a \cdot \frac{F_y}{u_y} \cdot x_1(t) + (1-a) \cdot F_y \cdot x_3(t) - f(t) \right] \\ \frac{1}{u_y} \cdot \left[\left(A - |x_3(t)|^n \cdot \left(\gamma \cdot sign\left(x_2(t) \cdot x_3(t) \right) + \beta \right) \right) \cdot x_2(t) \right] \end{Bmatrix} \tag{6}$$

The above system of three non-linear ordinary differential equations (ODEs) is solved numerically using Livermore stiff ODE integrator, which is based on the "predictor-corrector" method [16] or by employing 4th-5th order Runge-Kutta method. The latter is more efficient in terms of computational time and thus more suitable for stochastic optimization algorithms; the former is more accurate and can be used for comparison purposes.

2.2 Parameter constraints

Ma et al. have come to the conclusion that the parameters of the original Bouc-Wen model are functionally redundant [17]. Typically, identification is based on error minimization of a predicted history $\hat{y}\left(t \mid \mathbf{p}\right)$ as compared to a reference history $y(t)$. If the parameter redundancy is not treated, the same optimized predicted history can be produced from a multitude of parameter vectors **p**. Thus, the actual outcome of identification is dependent on factors such as the method employed, initial conditions, a priori knowledge of true parameter values, even sheer chance. As a consequence, the identified parameter vector may be totally erroneous when the excitation characteristics are different. Clearly, the aforementioned issue is critical with respect to the ability of the model to predict the response under an unknown excitation.

From the mathematical point of view, various parameters may be fixed in order to remove the redundancy of the model [17]. However, physical parameterization requires that parameter A is fixed to the value of unity:

$$A = 1 \tag{7}$$

Indeed, based on Eqs. (1) and (2), it can be shown that the initial stiffness k_i^* exhibited by a system is given by:

$$k_i^* = \left.\frac{dF}{du}\right|_{t=0} = a \cdot \frac{F_y}{u_y} + \left(1-a\right) \cdot \frac{F_y}{u_y} \cdot A \tag{8}$$

It is observed that only when Eq. (7) holds does k_i^* become equal to $k_i=F_y/u_y$, as implied by the formulation of the model in Fig. 1. It is stressed that this modification does not decrease in any way the ability of the model in simulating hysteresis. Henceforth, Eq. (7) is assumed to hold without further notification.

Another important issue is related to parameters β and γ. Suitably chosen values of these parameters lead to hysteretic loops with strain-hardening, as demonstrated by Wen [2]. However, these parameters are purely mathematical and do not have physical interpretation. However, early studies by Constantinou and Adnane [18] suggested imposing a certain constraint, viz. $A/(\beta+\gamma)=1$, to reduce the model to a viscoplastic formulation. Strain hardening can be achieved by more efficient techniques, such as the introduction of a dedicated spring. This can be controlled by physical parameters, such as the displacement at which strain hardening is initiated; an example of this approach can be found in Sivaselvan and Reinhorn [5]. Thus, the second constraint is as follows:

$$\beta + \gamma = 1 \tag{9}$$

In this work, the term "modified Bouc-Wen model" will refer to the modifications imposed by Eqs. (7) and (9). The former eliminates the parameter redundancy while the latter is related to physical parameterization issues. The modified model displays good mechanical properties and contains no parameters that are consistently insensitive [17]. Thus, the identification problem becomes unimodal and a suitable algorithm will be devised for this purpose.

3 IDENTIFICATION SCHEME

3.1 General

Parameter identification can be converted into an optimization problem by invoking a suitable objective function. Evolutionary Algorithms (EAs), which are inspired by natural selection and survival of the fittest, are considered to be amongst the most reliable and efficient methods for global optimization. They are able to provide near-optimum results by evolving a small population of candidate solutions. In particular, EAs are suitable for identification purposes for two main reasons: first, they rely on "payoff" data, i.e. not derivative data, which is very important for highly non-linear problems; second, they possess an inherent capability for massive parallel computing.

Invariably, EAs are characterized by the so-called anytime behavior; the development of the population's best individual shows rapid progress in the beginning, followed by gradual degradation until the point when evolution practically stops. Thus, it is not beneficial to let the EA evolve for too many generations, since progress will be limited and expensive [19]. The basic idea behind the proposed scheme is to take advantage only of the initial explosive part of evolution. Starting with very wide initial ranges for the parameters, so that inclusion of the optimal values is ensured, the EA is applied for a number of independent runs of few generations each, so as to provide a sufficient sample of the best parameter values. This sample is analyzed statistically using weight and truncation and new narrowed ranges for the parameters are calculated. This procedure is repeated until the ranges of all parameters are narrow enough, at which point identification stops. Thus, the algorithm is designed to focus in a single region of the search space and works very well when applied to the identification of the modified Bouc-Wen model, as will be demonstrated.

3.2 Objective function

In this study, the normalized Mean Square Error (MSE) of the predicted time history is used as objective function. In general, the discrete normalized form of the MSE of a predicted time history $\hat{y}(t\,|\,\mathbf{p})$ as compared to a reference time history $y(t)$ can be expressed as:

$$OF(\mathbf{p}) = \frac{\sum_{i=1}^{N}\left(y(t_i) - \hat{y}(t_i\,|\,\mathbf{p})\right)^2}{N \cdot \sigma_y^2} \cdot 100\% \tag{10}$$

Where, $\mathbf{p}$ is the parameter vector, $\sigma_y^{\,2}$ the variance of the reference time history and N the number of points used. The time history of the displacement and external force is used for force- and displacement-controlled experiments, respectively.

3.3 Evolutionary algorithm

Due to its modular structure, the proposed scheme can accommodate any EA. In this work, a recently introduced GA variant, called Sawtooth GA [14], was selected. Sawtooth GA uses variable population size and partial reinitialization in a synergistic way to enhance performance. This approach was compared to pure-GA and other GAs that use reinitialization of the population, such as the micro-GA, first suggested by Goldberg [20], with very good results. According to Sawtooth GA, the population size follows a predefined scheme (Fig. 2).

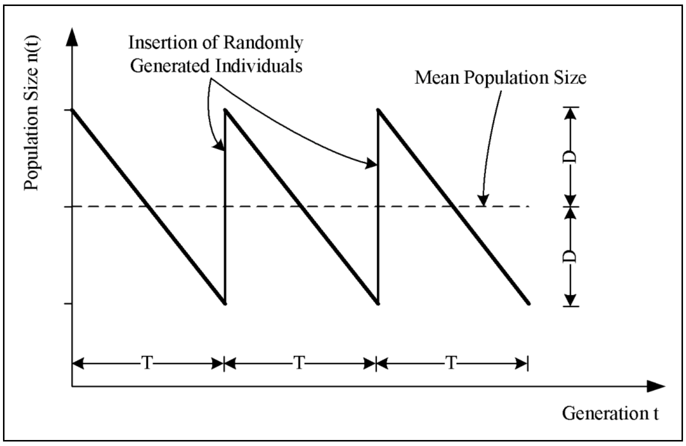

Fig. 2: Sawtooth population variation scheme

The scheme is characterized by amplitude D and period of variation T. Thus, at a specific generation t, the population size $n(t)$ is determined as:

$$n(t) = \text{int}\left\{N + D - \frac{2 \cdot D}{T-1} \cdot \left[t - T \cdot \text{int}\left(\frac{t-1}{T}\right) - 1\right]\right\} \tag{11}$$

Where, N is the mean population size. From parametric studies with a large testbed of unimodal and multimodal problems, it become evident that strong reinitialization of the population, i.e. large values of D/N, was beneficial. Moreover, moderate periods performed well, with values of T/N around 0.50.

3.4 Bounding

Let i, j be the enumerator of parameter and identification step, respectively. At each step j, the GA is applied for M_r independent runs of few generations each. These runs can be performed in parallel and the results can be collected by a host computer, thus reducing drastically the computational time. The results are sorted by the objective function value and the worse M_t ones are truncated. Let k be the enumerator of the remaining results, ranging between 1 and M_r - M_t. The weight of result k is given by:

$$w_k = \frac{\max_k (OF_k)}{OF_k} \tag{12}$$

Where, OF_k is the objective function value of result k. According to the above formulation, the worst result is assigned a weight of unity and the rest of the results are assigned weights greater than one. The weighted mean value of parameter i is calculated as follows:

$$m_i = \frac{\sum_{k=1}^{M_r - M_t} (w_k \cdot p_{ik})}{\sum_{k=1}^{M_r - M_t} (w_k)} \tag{13}$$

Where, p_{ik} is the value of parameter i of best individual k. The descriptive weighted standard deviation of parameter i is calculated as follows:

$$s_i = \sqrt{\frac{\sum_{k=1}^{M_r - M_t} \left(w_k \cdot \left(p_{ik} - m_i \right)^2 \right)}{\sum_{k=1}^{M_r - M_t} \left(w_k \right)}} \tag{14}$$

The new trial upper and lower bounds of parameter i are defined symmetrically around the weighted mean value m_i as follows:

$$\begin{aligned} \overline{u}_{i,j+1} &= m_i + q \cdot s_i \\ \overline{l}_{i,j+1} &= m_i - q \cdot s_i \end{aligned} \tag{15}$$

Where, q is a real parameter with a typical value of 3.0. Finally, the new range of parameter i is given by:

$$\left[l_{i,j+1}, u_{i,j+1} \right] = \left[l_{ij}, u_{ij} \right] \cap \left[\overline{l}_{i,j+1}, \overline{u}_{i,j+1} \right] \tag{16}$$

Two important points are noted; first, based on Eq. (16), the parameter ranges are not allowed to expand. If the specific parameter is insensitive then the results will be scattered almost uniformly in the allowed range. Provided that the statistical sample is sufficient, the standard deviation will be large and the range will not be altered. In this way, narrowing of a range occurs only when there is enough information to justify it.

Another important point is that the proposed method is very safe when the optimum parameter value lies at one end of the range. This is because, in general, the weighted mean value will lie close to the same end; since the new trial range is formed symmetrically around it, the optimum value will remain within boundaries.

4 IDENTIFICATION

In this work, the oscillating mass is the only parameter that is considered known or measurable with sufficient accuracy. Nonetheless, the performance, even the success, of any identification scheme depends heavily on the input-output data. These should be maximally informative, in the sense that they should encapsulate all hysteretic characteristics. Moreover, parameter sensitivity depends not only on the mathematical formulation of a model but also on the excitation.

Identification of hysteretic, i.e. rate-independent characteristics can be performed with a simple periodic excitation of few cycles; in this case, viscous damping is either omitted or considered known. To illustrate this, three cases of 3-period displacement-controlled sinusoidal experiments, namely cases 1a to 1c, are considered in Fig. 3. The amplitude is equal to 0.5, 2.0 and 7.0 times the yield displacement, respectively. In addition, the El Centro excitation was considered, as case 1d. The true parameter values, initial side constraints and mean results of ten independent runs are summarized in Table 1, while the comparative performance is shown in Fig. 4.

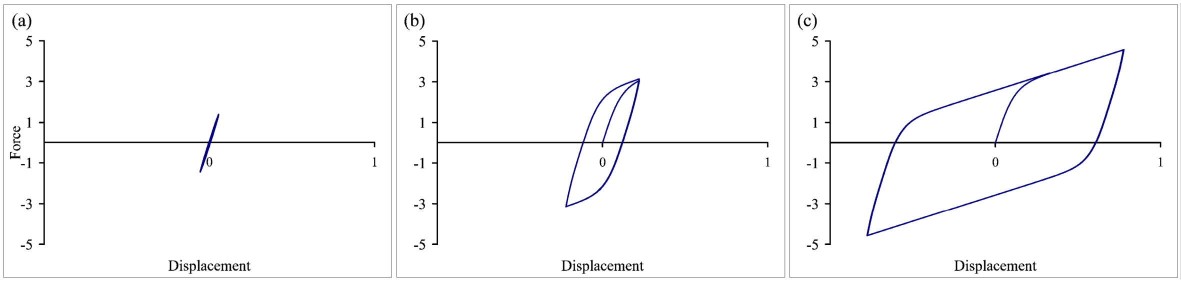

Fig. 3: Hysteretic loops for sinusoidal displacement with umax / uy (a) = 0.5, (b) 2.0, (c) 7.0.

	γ	n	a	F_y	u_y	c
True value	0.9000	2.0000	0.1000	2.8600	0.1110	0.0000
Initial lower bound	0.0000	1.0000	0.0000	0.1000	0.0100	-
Initial upper bound	1.0000	10.0000	1.0000	10.0000	1.0000	-
1a (u_{max} = 0.5 uy)	0.8856	1.9562	0.1075	2.8987	0.1123	-
1b (u_{max} = 2.0 uy)	0.8991	1.9986	0.1000	2.8597	0.1110	-
1c (u_{max} = 7.0 uy)	0.8676	1.9220	0.0987	2.8552	0.1093	-
1d (El Centro)	0.9000	2.0000	0.1000	2.8600	0.1110	-

Table 1: Parameter values, initial side constraints and final results (viscous damping known)

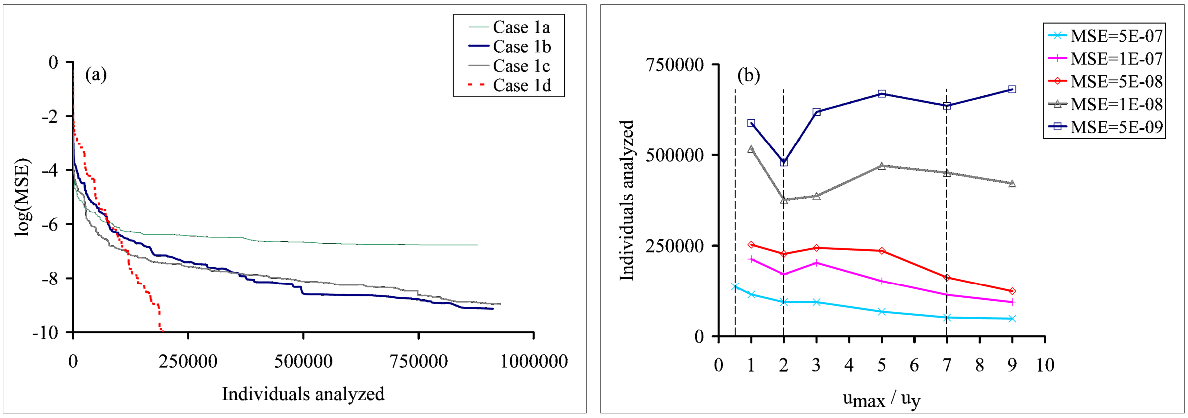

Fig. 4: Comparative performance (viscous damping known).

In the case of El Centro excitation (case 1d), progress is rapid and exact parameter values, i.e. to four decimal digits, are readily obtained. For the cases of sinusoidal displacement, the first response is almost linear and does not contain information on the post-elastic regime; as a result, identification fails to progress (Fig. 4a). In case 1c, the response is strongly non-linear and the best initial performance is observed (Fig. 4a,b). This is because most of the information refers to the post-elastic regime and the corresponding bilinear skeleton is identified quickly. For the opposite reason, however, the sensitivity of the parameters controlling the transition between branches is small. In this work, the stop criterion for identification refers to narrowing of parameter ranges rather than reaching a MSE threshold; thus, the best overall performance is exhibited by case 1b, in which the system just enters the post-elastic region and the sensitivity of all parameters is on the same level (Fig. 4a,b).

When viscous damping is unknown, identification with a single sinusoidal experiment fails occasionally by predicting values of viscous damping coefficient greater than the true one. This is to be expected, since under simple harmonic excitation all damping can be attributed to viscous-type effects of a linear system. The concept of equivalent viscous damping has long been used in structural analysis in the past because of its simplicity. However, it is known that this transformation is valid only for the specific amplitude and frequency; moreover, the equivalent viscous damping factors are strongly dependent on amplitude [21]. Therefore, this problem can be readily addressed by combining two experiments of different amplitude in one e.g. combining cases 1b and 1c. In this case, the mean normalized MSE can be used as objective function and identification with unknown viscous-type damping is always successful. Three cases, namely cases 2a to 2c, are considered. These are, respectively, a failed attempt using experiment 1b only, a combination of cases 1b and 1c and the El Centro excitation. The true parameter values, initial side constraints and identification results are summarized in Table 2. The hysteretic loop of case 2a, as compared to the true system response, is shown in Fig. 5a. Exact parameter values are obtained for both cases 2b and 2c; however, it is observed that the former case, i.e. the combination of two sinusoidal experiments, exhibits roughly 3 times better performance (Fig. 5b).

	γ	n	a	F_y	u_y	c
True value	0.9000	2.0000	0.1000	2.8600	0.1110	5.4292
Initial lower bound	0.0000	1.0000	0.0000	0.1000	0.0100	0.0000
Initial upper bound	1.0000	10.0000	1.0000	10.0000	1.0000	100.0000
2a (umax = 2 uy)	0.6211	1.5397	0.1736	2.5542	0.1013	6.4835
2b (umax = 2; 7 uy)	0.9000	2.0000	0.1000	2.8600	0.1110	5.4292
2c (El Centro)	0.9000	2.0000	0.1000	2.8600	0.1110	5.4292

Table 2: Parameter values, initial side constraints and final results (viscous damping unknown)

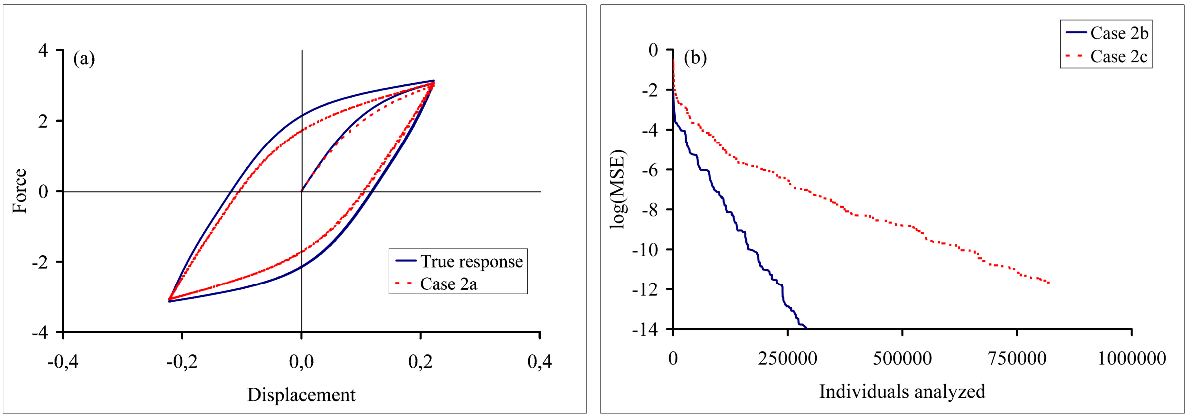

Fig. 5: (a) Case 2a as compared to true response (b) performance comparison of cases 2b and 2c.

5 NOISE

In reality, experimental data are always corrupted by noise of various origins. The proposed scheme was used to identify hysteretic systems with noise-corrupted data that are produced in the following manner:

$$\bar{y}(t_i) = (1 + \varepsilon \cdot r_i) \cdot y(t_i) \tag{17}$$

Where, r_i's are a sequence of random variables with uniform distribution in the interval $(-1,1)$ and parameter ε is the noise to signal ratio (NSR). Three levels of noise were considered, with NSR equal to 1%, 5% and 10%. The parametric configuration G2 and the initial side constraints of Table 2 were used. The oscillating mass was equal to 28.6 and the El Centro accelerogram was used in all cases. No noise pre-filtering was applied.

The mean best parameter values, summarized in Table 3, show that the proposed scheme is insensitive to noise. The bounding progress of the most insensitive parameter, i.e. parameter γ, is shown in Fig. 6. It is observed that faster convergence occurs for lower levels of noise contamination.

	γ	n	a	F_y	u_y	c
True value	0.9000	2.0000	0.1000	2.8600	0.1110	5.4292
NSR = 1%	0.9010	1.9998	0.1000	2.8608	0.1110	5.4261
NSR = 5%	0.9019	2.0096	0.1001	2.8569	0.1110	5.4369
NSR = 10%	0.8993	1.9953	0.0995	2.8642	0.1111	5.4014

Table 3: Identification results from noise-corrupted data

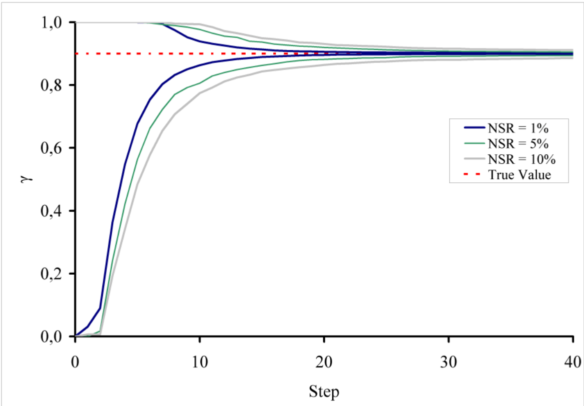

Fig. 6: Bounding progress of parameter γ for noise-corrupted data

6 CONCLUSIONS

In this work, a new stochastic identification method is presented. The scheme consists of a new GA variant, called Sawtooth GA and a Bounding process. The initial parameter ranges are very wide so that inclusion of the optimal values is ensured. Mass is the only system property that is considered known; stiffness and viscous-type damping characteristics are identified. Progress occurs as the algorithm continuously focuses in better regions of the search space by narrowing the ranges of parameters. Instead of being trapped into local optima, the narrowing process is suspended when statistic analysis of the data does not justify progress.

Due to its modular structure, the scheme can accommodate any evolutionary algorithm. Moreover, it lends itself very easily to massive parallel computing. Assuming that the network is consisted of similar computer systems and the overhead from data transfer is negligible, the time required is inversely analogous to the number of computers used.

Numerous experiments show that the proposed scheme is extremely robust in identifying systems with noise-free or noise-corrupted data. The efficiency of the method is investigated with respect to experiment design and whether viscous-type effects are identified or not. Although a limited set of data is presented, many experiments have been conducted that support any observations made herein.

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