

MYSPEC: Educational Software for Structural Dynamics and Hysteretic Systems

A. E. Charalampakis and V. K. Koumousis
Institute of Structural Analysis and Aseismic Research,
National Technical University of Athens,
NTUA, Zografou Campus GR-15773, Athens, Greece

Abstract

This paper presents the basic features of mySpec, a computer program that has been designed to enhance learning in subjects that cover structural dynamics, earthquake engineering and hysteretic systems in graduate and post-graduate level. The program features a fully graphical interface with many interactive capabilities and covers the following topics: linear elastic analysis, linear elastic response spectra, non-linear analysis (bilinear) and Bouc-Wen hysteretic systems with arbitrary excitation force, either alone or combined with an earthquake accelerogram. Viscous damping can be used in all cases. The program is freely distributed through the Internet.

Keywords: Educational software, structural dynamics, earthquake engineering, hysteretic systems, Bouc-Wen.

1 Introduction

Deeper understanding of the response of multi-degree-of-freedom structures under dynamic excitation is of major importance for the Structural Engineer. This knowledge is not always intuitive and may be assisted considerably by means of interactive software that provides results for simplified systems, such as Single-Degree-Of-Freedom (SDOF) systems or Two-Degree-Of-Freedom (2DOF) systems.

MySpec was written to provide the undergraduate and postgraduate student a user-friendly computational tool that is capable of analyzing simple systems with various simple or more advanced models and to provide results in a visual and comprehensive way, thus revealing both qualitative and quantitative characteristics of the response.

2 General features

2.1 Earthquake accelerograms

MySpec features an accelerogram library which can be loaded with earthquake accelerograms. Each accelerogram can be modified, i.e. stretched with respect to amplitude or duration, thus allowing the student to experiment with the response of these systems.

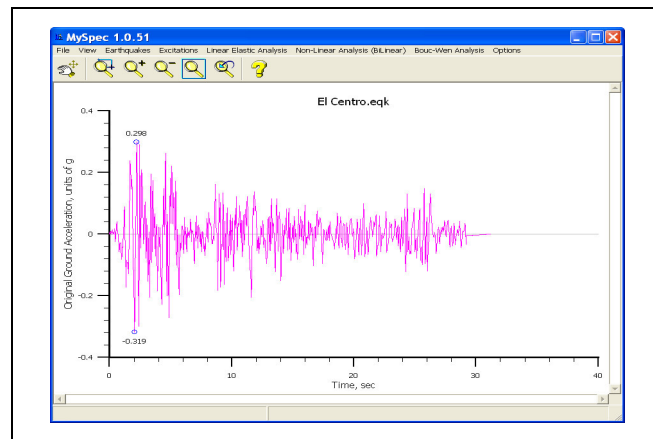


Figure 1: Earthquake accelerograms in mySpec.

2.2 Arbitrary force excitation

MySpec also features an arbitrary force library which can be loaded with force histories. Similarly, each force history can be stretched with respect to amplitude or duration. Moreover, the force excitation can be used either alone or combined with an earthquake accelerogram.

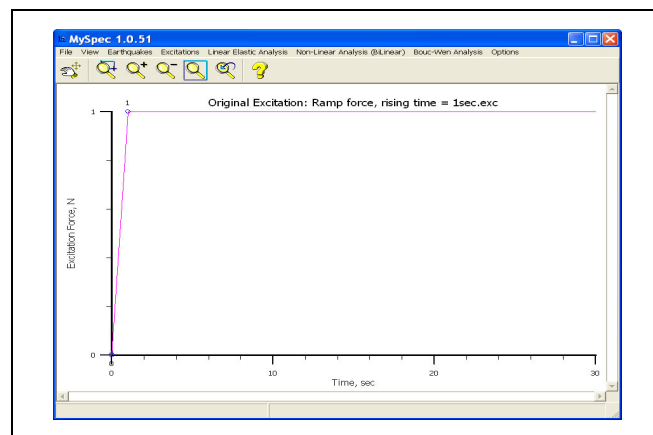


Figure 2: Arbitrary force excitation in mySpec.

2.3 Export results

MySpec uses double precision arithmetic and it is capable of exporting the results in a standard fixed-length ASCII file. The data is therefore available for further processing by other software, such as spreadsheet software.

2.4 Simulation of the response of SDF systems

MySpec is capable of simulating the response of SDF systems in a visual and comprehensive way. The simulation form includes a sketch of the system as a lumped mass on top of a massless column. If the excitation is an earthquake then the motion of the ground is included. The user can select the simulation speed, has direct control over the simulation progress while any diagram of interest can be plotted on the screen as the simulation progresses, as for example in Fig.3.

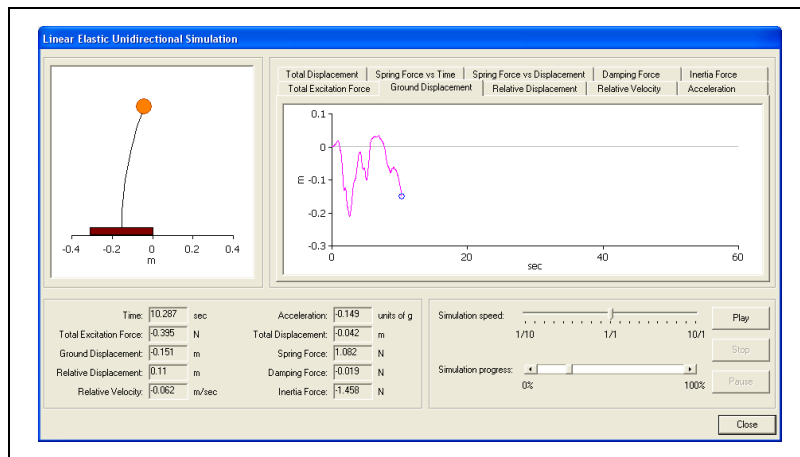


Figure 3: Simulation of the response of SDF systems.

2.5 Simulation of the response of 2DoF systems

MySpec is also capable of simulating the response of 2DoF systems. Similarly, the system is sketched as a lumped mass on top of a massless column and if the excitation is an earthquake then the motion of the ground is included. The user can select the simulation speed, has direct control over the simulation progress, while any diagram of interest can be plotted on screen either in X or Y direction as presented in Fig. 4.

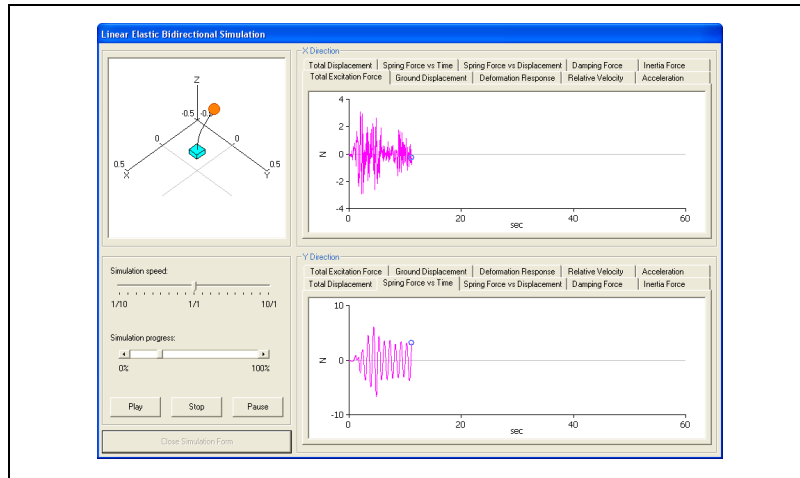


Figure 4: Simulation of the response of 2DoF systems.

2.6 Newmark's method

In many cases, the response of the system is calculated with the well-known Newmark's method. In 1959, N. M. Newmark developed a family of time stepping methods, based on the following equations:

$$\dot{u}_{i+1} = \dot{u}_i + [(1-\gamma) \cdot \Delta t] \cdot \ddot{u}_i + (\gamma \cdot \Delta t) \cdot \ddot{u}_{i+1}$$

$$u_{i+1} = u_i + (\Delta t) \cdot \dot{u}_i + [(0.5 - \beta) \cdot (\Delta t)^2] \cdot \ddot{u}_i + [\beta \cdot (\Delta t)^2] \cdot \ddot{u}_{i+1}$$

Typical selection for γ is $1/2$. For β a selection in the range $1/6 \leq \beta \leq 1/4$ is satisfactory. The following set of values is used for the average acceleration:

$$\gamma = 1/2$$

$$\beta = 1/4$$

Another set of values is used for the linear acceleration:

$$\gamma = 1/2$$

$$\beta = 1/6$$

These set of values are directly related to the assumption of the variation of the acceleration during the time step.

2.7 ODE integration

For the analysis of Bouc-Wen type hysteretic systems, mySpec uses a robust Livermore stiff ODE integrator which is based on a “predictor-corrector” method [4].

2.8 Analysis options

The analysis options are conveniently specified in one form, which appears by selecting “Analysis” under the “Options” menu of the main form.

The screenshot shows a dialog box titled "Analysis options" with a blue header. It contains several sections with input fields and checkboxes. The "Time step - use the minimum of:" section has three checked options: "Excitation's time step /" (value: 1), "Period /" (value: 25), and "Absolute value:" (value: 0.02). The "Additional duration" section has a field for "Calculate the response for an additional duration of:" (value: 20) and "periods (Tn)". The "Newmark parameters" section has "Beta:" (value: 0.166666666666667), "Gamma:" (value: 0.5), and "Method:" (dropdown: Linear Acceleration). The "Bouc Wen parameters" section has "Alpha:" (value: 1), "Beta:" (value: 0.1), and "Gamma:" (value: 0.9). At the bottom are "Load Default Values", "OK", and "Cancel" buttons.

Figure 5: Analysis options.

In the “Time step – use the minimum of” frame, the user can set restrictions on the maximum value of the time step used in the calculations. In general, the user should not modify these settings as this may affect the accuracy of calculations.

In the “Additional duration” frame, the user can select the additional duration for which the program should calculate the response. This period of time corresponds to free vibration after the end of the excitation and provides also very interesting results. All graphs are plotted with magenta colour for the forced motion, followed by green colour for the duration of the free vibration.

In the “Newmark parameters” frame, the user can select the parameters β and γ of Newmark’s method.

In the “Bouc Wen parameters” frame, the user can select the parameters A, β, γ of Bouc Wen model [2], [3].

3 Analysis

3.1 Linear elastic analysis for an SDF system

3.1.1 Model

The equation of motion for a viscously damped linear elastic SDF system is the following:

$$m \cdot \ddot{u} + c \cdot \dot{u} + k \cdot u = p(t)$$

Where u is the relative displacement of the system, $m \cdot \ddot{u}$ is the inertia force, $c \cdot \dot{u}$ is the damping force, c is the viscous damping coefficient, $k \cdot u$ is the spring force and $p(t)$ is the external dynamic force [1].

In case of an earthquake, the external dynamic force is $p(t) = -m \cdot \ddot{u}_g(t)$, where $\ddot{u}_g(t)$ is the ground acceleration. The ground displacement $u_g(t)$ is a function of time, whereas the total (or absolute) displacement $u_t(t)$ is the sum of the ground displacement $u_g(t)$ and the relative displacement $u(t)$ of the system at all times.

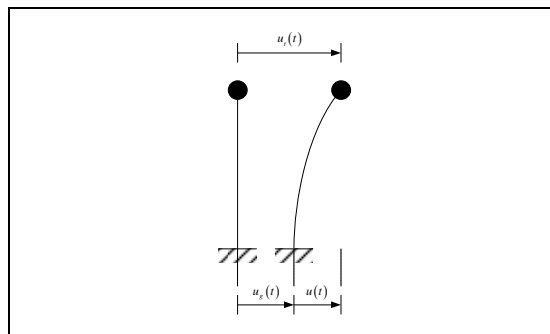


Figure 6: Total displacement of a SDOF system.

The linear elastic response is calculated following the well-known Newmark's Method.

3.1.2 Input data

In order to input the data, select "Single Unidirectional > Options" from the "Linear Elastic Analysis" menu. The following form will appear:

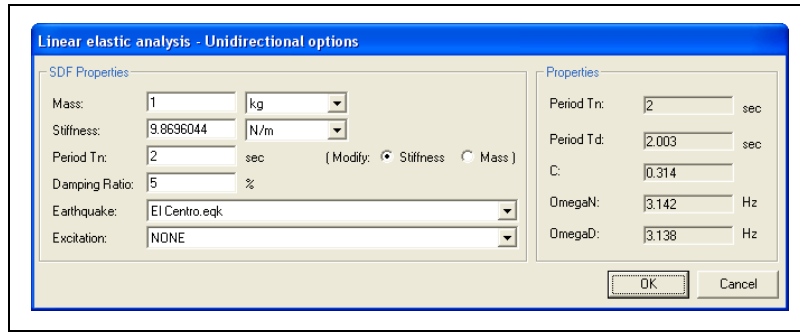


Figure 7: Input data for an elastic SDF system.

The data required is the following:

- Mass: The mass of the SDF system.
- Stiffness: The stiffness of the SDF system.
- Period: The natural period T_n of the system in seconds, which is calculated as $T_n = 2 \cdot \pi \cdot \sqrt{m/k}$.
- Damping ratio: The damping ratio ζ of the system, in percentage (%). The damping ratio (or fraction of critical damping) is defined as $\zeta = c/c_{cr}$. The critical damping coefficient is defined as $c_{cr} = 2 \cdot m \cdot \omega_n = 2 \cdot m \cdot \sqrt{k/m}$, where ω_n is the natural frequency. The critical damping coefficient is used in viscously damped vibrations. For example, the equation governing viscously damped free vibration of a SDF system is $\ddot{u} + 2 \cdot \zeta \cdot \omega_n \cdot \dot{u} + \omega_n^2 \cdot u = 0$.
- Earthquake: In addition to or separately from the excitation, you can select the desired earthquake from the drop-down list. Note that the modified form of the earthquake is used in the calculations.
- Excitation: In addition to or separately from the earthquake, you can select the desired excitation from the drop-down list. Note that the modified form of the excitation is used in the calculations.

The “properties” frame contains the following data:

- Period T_n : The natural period of the system in seconds, which is calculated as $T_n = 2 \cdot \pi \cdot \sqrt{m/k}$.
- Period T_d : The natural period of damped vibration in seconds, which is related to T_n by: $T_d = T_n / \sqrt{1 - \zeta^2}$.
- c : The damping coefficient, which is related to the damping ratio ζ by: $c = 2 \cdot m \cdot \omega_n \cdot \zeta$.

- OmegaN: The natural frequency ω_n of the SDF system, which is given by:

$$\omega_n = \sqrt{k/m} .$$
- OmegaD: The natural frequency of damped vibration ω_d of the SDF system, which is related to the undamped natural frequency ω_n by: $\omega_d = \omega_n \cdot \sqrt{1 - \zeta^2} .$

3.2 Linear elastic analysis for a 2DoF system

3.2.1 Model

For the case of 2DoF systems, the equations of motion are uncoupled and have the same form as the case of the SDF system while Newmark's method is used for the calculation of the response.

3.2.2 Input data

In order to input the data, select “Single Bidirectional > Options” from the “Linear Elastic Analysis” menu. The following form will appear:

Figure 8: Input data for an elastic 2DoF system.

The data required for each of the two directions is described in detail for the case of SDF systems.

3.3 Elastic response spectrum

3.3.1 Calculations

Given a range of periods for a SDOF system and the time step, mySpec can produce the elastic response spectrum of various response parameters for a specific excitation and / or earthquake. The response is calculated using Newmark's method for each period. The mass of the SDOF system is needed in case of force excitations. Also, viscous damping may be taken into account.

The program may trim the very small periods or use a minimum time step because the iterative process is time consuming; for each period the full response of the SDOF system must be calculated so that the peak values can be stored. MySpec can produce response spectra for the following quantities:

- Deformation: The peak value of deformation D for each period.
- Pseudo velocity: The peak value of the pseudo velocity V which is defined as $V = \omega_n \cdot D$, where ω_n is the natural frequency and D is the peak deformation of the same system. Pseudo velocity has units of velocity and it is related to the peak value of the strain energy E_{s0} stored in the system during the excitation, given by $E_{s0} = mV^2/2$.
- Pseudo acceleration: The peak value of the pseudo acceleration A which is defined as $a = \omega_n^2 \cdot D$, where ω_n is the natural frequency and D is the peak deformation of the same system. Pseudo acceleration has units of acceleration and it is related to the peak value of base shear V_{b0} or the peak value of the equivalent static force f_{s0} , given by $V_{b0} = f_{s0} = m \cdot A$.
- Relative Velocity: The peak value of the relative velocity.
- Acceleration: The peak value of the acceleration.

3.3.2 Input data

In order to input the data, select “Response Spectrum > Options” from the “Linear Elastic Analysis” menu. The following form will appear:

The screenshot shows a dialog box titled "Linear Spectral Analysis - Options". It contains the following fields and controls:

- Minimum period: 0 sec
- Maximum period: 3 sec
- Period time step: 0.05 sec
- Damping: 2 %
- Mass: 0 kg
- Earthquake: El Centro.eqk
- Excitation: NONE
- Buttons: OK, Cancel

Figure 9: Input data for an elastic response spectrum.

The data required is the following:

- Minimum period: The minimum period of the range, in seconds.
- Maximum period: The maximum period of the range, in seconds.
- Period time step: The time step, in seconds.
- Damping: The damping ratio ζ of the system, in percentage (%). The damping ratio (or fraction of critical damping) is defined as $\zeta = c/c_{cr}$. The critical damping coefficient is defined as $c_{cr} = 2 \cdot m \cdot \omega_n$, where $\omega_n = \sqrt{k/m}$ is the natural frequency.
- Mass: The mass of the SDF system. This is required in case of force excitations.
- Earthquake: In addition to or separately from the excitation, you can select the desired earthquake from the drop-down list. Note that the modified form of the earthquake is used in the calculations
- Excitation: In addition to or separately from the earthquake, you can select the desired excitation from the drop-down list. Note that the modified form of the excitation is used in the calculations

3.4 Non-linear analysis (bilinear model) for an SDOF system

3.4.1 Model

MySpec evaluates the nonlinear response based on a generic bilinear model. A typical bilinear force - displacement model is shown below:

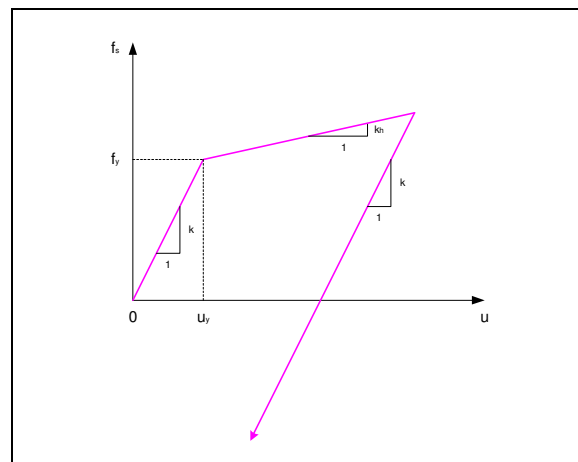


Figure 10: Generic bilinear model.

The system is linearly elastic with stiffness k up to the yield force. For displacement bigger than u_y the system responds with a constant hardening stiffness

k_h . In the unloading branch, the system regains its initial stiffness k . The yield strength f_y and the hardening stiffness k_h may be different for the positive and negative directions of loading. Also, if the system has yielded and is unloaded, the new elastic branch has the same yield strength range as the initial values i.e. $f_{y.pos} + |f_{y.neg}|$ but may be displaced in the positive or the negative direction because of the hardening stiffness.

The response is calculated with the well-known Newmark Method. In general, Newmark's method is very satisfactory in terms of accuracy. Since the time step is constant, two are the main sources of error:

- The tangent stiffness is used instead of the (actual) secant stiffness in the calculation of the incremental resisting force. The secant stiffness cannot be used because it is not known.
- The detection of the transitions in the force – deformation relationship is inaccurate. This inevitably leads to error accumulation.

MySpec addresses these errors by using a small time step which can be modified by the user and by modifying the time step at the transitions from the elastic to inelastic branch and vice versa. In the first case i.e. the transition from the elastic to the inelastic branch, the condition is that the resisting force equals the yield strength: $f_s = f_y$, whereas in the second case i.e. the transition from the loading to the unloading branch, the condition is that the velocity is zero: $\dot{u} = 0$. When a transition is detected, the time step is continuously divided by two, in order to detect the transition point accurately. When the corresponding condition is met to a certain accuracy, the algorithm continues to the next time step.

3.4.2 Input data

In order to use the bilinear model, select “Options” from the “Non Linear Analysis (Bilinear)” menu. The following form will appear:

Non linear analysis - Unidirectional options		
SDF Properties		
Mass:	10	kg
Stiffness:	98.69604401	N/m
Period Tn:	2	sec (Modify: <input checked="" type="radio"/> Stiffness <input type="radio"/> Mass)
Positive Yield:	5	N
Positive Hardening Stiffness:	2	N/m
Negative Yield:	5	N
Negative Hardening Stiffness:	10	N/m
Damping Ratio:	2	%
Earthquake:	EI Centro. eqk	
Excitation:	NONE	
<input type="button" value="OK"/> <input type="button" value="Cancel"/>		

Figure 11: Input data for the bilinear model.

In the “SDF Properties” frame, the following data is required:

- Mass: The mass of the SDF system.
- Stiffness: The stiffness of the SDF system
- Period: The natural period T_n of the system in seconds, which is calculated as

$$T_n = 2 \cdot \pi \cdot \sqrt{m/k}$$
- Positive yield: The positive yield strength of the system.
- Positive hardening stiffness: The positive hardening stiffness of the system
- Negative yield: The negative yield strength of the system (use positive values).
- Negative hardening stiffness: The negative hardening stiffness of the system (use positive values).
- Damping: The damping ratio ζ of the system, in percentage (%). The damping ratio (or fraction of critical damping) is defined as $\zeta = c/c_{cr}$. The critical damping coefficient is defined as $c_{cr} = 2 \cdot m \cdot \omega_n$, where $\omega_n = \sqrt{k/m}$ is the natural frequency.
- Earthquake: In addition to or separately from the excitation, you can select the desired earthquake from the drop-down list. Note that the modified form of the earthquake is used in the calculations
- Excitation: In addition to or separately from the earthquake, you can select the desired excitation from the drop-down list. Note that the modified form of the excitation is used in the calculations

3.5 Non-linear analysis (Bouc Wen model) for a 2DoF system

3.5.1 Introduction

MySpec evaluates the nonlinear response for a 2DoF system based on a generic Bouc-Wen Model. This hysteretic or memory – dependent model is very popular because of its versatility and simplicity; it is a very concise model governed by a single differential equation. This was first introduced by Bouc in 1967 [2]. In 1976, Wen [3], extended the model and demonstrated its versatility by producing a variety of hysteretic patterns.

3.5.2 SDOF System

The restoring force of a SDOF system can be written as:

$$F(t) = a \cdot \frac{F_y}{u_y} \cdot u(t) + (1-a) \cdot F_y \cdot z(t) \quad (1)$$

where, F_y is the yield force, u_y is the yield displacement, a is the ratio of post-yield to pre-yield (elastic) stiffness and $z(t)$ is a dimensionless hysteretic parameter satisfying the following nonlinear differential equation:

$$\dot{z}(t) = \frac{1}{u_y} \left[A - |z(t)|^n \cdot (\gamma \cdot \text{sign}(\dot{u}(t) \cdot z(t)) + \beta) \right] \cdot \dot{u}(t) \quad (2)$$

where, A, β, γ, n are dimensionless quantities controlling the shape of the hysteretic loop. In particular, small value of n implies smooth transition from the elastic to the plastic branch, whereas a large value of n ($n > 10$) results into an abrupt transition and the model is similar to a bilinear one. The equation of motion for a SDOF system with external viscous damping c is given as:

$$m \cdot \ddot{u}(t) + c \cdot \dot{u}(t) + F(t) = f(t) \quad (3)$$

where, $u(t)$ is the displacement, $F(t)$ is the restoring force, $f(t)$ is the excitation force. Substituting (1) into (3) we obtain:

$$m \cdot \ddot{u}(t) + c \cdot \dot{u}(t) + a \cdot \frac{F_y}{u_y} \cdot u(t) + (1-a) \cdot F_y \cdot z(t) = f(t) \quad (4)$$

Equations (2) and (4) are transformed into a state-space form as follows:

$$\begin{cases} x_1(t) = u(t) \\ x_2(t) = \dot{u}(t) \\ x_3(t) = z(t) \end{cases} \quad (5)$$

$$\begin{cases} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{cases} = \begin{cases} x_2(t) \\ -\frac{1}{m} \cdot \left[c \cdot x_2(t) + a \cdot \frac{F_y}{u_y} \cdot x_1(t) + (1-a) \cdot F_y \cdot x_3(t) - f(t) \right] \\ \frac{1}{u_y} \cdot \left[\left(A - |x_3(t)|^n \cdot (\gamma \cdot \text{sign}(x_2(t) \cdot x_3(t)) + \beta) \right) \cdot x_2(t) \right] \end{cases} \quad (6)$$

The above system of three first order non-linear ODEs is solved numerically following Livermore stiff ODE integrator which is based on a ‘‘predictor-corrector’’ method [4].

3.5.3 2DoF Model

Extending the above equations for a 2DoF system, the equations of motion in two directions x and y are two ODEs of second order in time:

$$\begin{aligned} & \begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix} \cdot \begin{Bmatrix} \ddot{u}_x \\ \ddot{u}_y \end{Bmatrix} + \begin{bmatrix} c_x & 0 \\ 0 & c_y \end{bmatrix} \cdot \begin{Bmatrix} \dot{u}_x \\ \dot{u}_y \end{Bmatrix} + \begin{bmatrix} a_x \cdot F_{y,x}/u_{y,x} & 0 \\ 0 & a_y \cdot F_{y,y}/u_{y,y} \end{bmatrix} \cdot \begin{Bmatrix} u_x \\ u_y \end{Bmatrix} + \\ & \begin{bmatrix} (1-a_x) \cdot F_{y,x} & 0 \\ 0 & (1-a_y) \cdot F_{y,y} \end{bmatrix} \cdot \begin{Bmatrix} z_x \\ z_y \end{Bmatrix} = \begin{Bmatrix} f_x \\ f_y \end{Bmatrix} \end{aligned} \quad (7)$$

These equations are coupled through the dimensionless hysteretic variables $z_x(t)$, $z_y(t)$ which are governed by the system of non-linear equations:

$$\begin{aligned} & \begin{Bmatrix} \dot{z}_x \cdot u_{y,x} \\ \dot{z}_y \cdot u_{y,y} \end{Bmatrix} = A \cdot \begin{Bmatrix} \dot{u}_x \\ \dot{u}_y \end{Bmatrix} - \\ & \begin{bmatrix} z_x^2 \cdot (\gamma \cdot \text{sign}(\dot{u}_x \cdot z_x) + \beta) & z_x \cdot z_y \cdot (\gamma \cdot \text{sign}(\dot{u}_y \cdot z_y) + \beta) \\ z_x \cdot z_y \cdot (\gamma \cdot \text{sign}(\dot{u}_x \cdot z_x) + \beta) & z_y^2 \cdot (\gamma \cdot \text{sign}(\dot{u}_y \cdot z_y) + \beta) \end{bmatrix} \cdot \begin{Bmatrix} \dot{u}_x \\ \dot{u}_y \end{Bmatrix} \end{aligned} \quad (8)$$

These equations were developed by Park et al. [5]. For the above system of equations $n=2$. Equations (7) and (8) are converted in state space form by introducing additional equations as follows:

$$\begin{aligned} & \begin{Bmatrix} x_1 = u_x \\ x_2 = \dot{u}_x \\ x_3 = z_x \\ x_4 = u_y \\ x_5 = \dot{u}_y \\ x_6 = z_y \end{Bmatrix} \end{aligned} \quad (9)$$

$$\begin{cases} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \end{cases} = \begin{cases} x_2 \\ -\frac{1}{m} \cdot \left[c_x \cdot x_2 + a_x \cdot \frac{F_{y,x}}{u_{y,x}} \cdot x_1 + (1-a_x) \cdot F_{y,x} \cdot x_3 - f_x \right] \\ \frac{[A \cdot x_2 - x_3^2 \cdot (\gamma \cdot \text{sign}(x_2 \cdot x_3) + \beta) \cdot x_2 - x_3 \cdot x_6 \cdot (\gamma \cdot \text{sign}(x_5 \cdot x_6) + \beta) \cdot x_5]}{u_{y,x}} \\ x_5 \\ -\frac{1}{m} \cdot \left[c_y \cdot x_5 + a_y \cdot \frac{F_{y,y}}{u_{y,y}} \cdot x_4 + (1-a_y) \cdot F_{y,y} \cdot x_6 - f_y \right] \\ \frac{[A \cdot x_5 - x_3 \cdot x_6 \cdot (\gamma \cdot \text{sign}(x_2 \cdot x_3) + \beta) \cdot x_2 - x_6^2 \cdot (\gamma \cdot \text{sign}(x_5 \cdot x_6) + \beta) \cdot x_5]}{u_{y,y}} \end{cases} \quad (10)$$

The above system of six first order non-linear ODEs is solved numerically following Livermore stiff ODE integrator which is based on a ‘‘predictor-corrector’’ method [4].

3.5.4 Applications

This model is extremely useful for the investigation of the dynamic behaviour of hysteretic Lead Rubber Bearing (LRB) and Friction Pendulum Systems (FPS); it provides a unified base for the analyses of both types of isolators [6].

For a Lead Rubber Bearing (LRB) isolator, the restoring force is given by equation (1). The behaviour of the mass isolator in two directions is given by equation (7), (8).

On the other hand, a Coulomb friction sliding system requires multiple stick – slip conditions that result into a complicated system of equations. However, a modified viscoplasticity model leads to a convenient formulation that describes accurately the behaviour of a sliding system, especially for Teflon – Stainless Steel interfaces, where the coefficient of friction increases with velocity. In this case, the friction force is determined as:

$$F(t) = \mu_s \cdot m \cdot g \cdot z(t) \quad (11)$$

with:

$$\mu_s = f_{\max} - \Delta f \cdot \exp(-a \cdot |\dot{u}(t)|) \quad (12)$$

where, f_{\max} is the coefficient at a large velocity of sliding, Δf is the difference between the coefficient of friction at a large and a very low velocity of sliding and a

is a constant. Parameters f_{\max} and Δf are generally dependent on bearing pressure, whereas a is nearly independent of pressure.

The dimensionless quantity $z(t)$ follows again equation (2) and controls the stick – slip conditions. For slip conditions, $z(t) = \pm z_{\max}$, while for stick conditions (elastic behaviour) $|z(t)| < z_{\max}$ depends on the Bouc-Wen parameters A, β, γ and should be equal to ± 1 .

Considering sliding on a spherical surface alters the above behaviour by adding the pendulum effect in the second equation of state space ODE system. For small values of $u(t)/R$ the restoring force of the pendulum effect may be expressed by the term $-m \cdot g \cdot u(t)/R$:

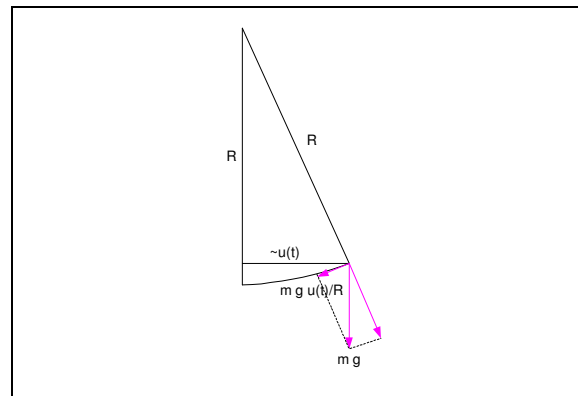


Figure 12: Restoring force of the pendulum effect.

3.5.5 Input data

In order to input the data, select “Single Bidirectional > Options” from the “Linear Elastic Analysis” menu. The following form will appear:

Figure 13: Input data for an elastic 2DoF system.

The data required for each of the two directions is described in detail for the case of SDF systems.

3.3 Elastic response spectrum

3.3.1 Calculations

Given a range of periods for a SDOF system and the time step, mySpec can produce the elastic response spectrum of various response parameters for a specific excitation and / or earthquake. The response is calculated using Newmark's method for each period. The mass of the SDOF system is needed in case of force excitations. Also, viscous damping may be taken into account.

The program may trim the very small periods, or use a minimum time step because the iterative process is time consuming; for each period the full response of the SDF system must be calculated so that the peak values can be stored.

MySpec can produce response spectra for the following quantities:

- Deformation: The peak value of deformation D for each period.
- Pseudo velocity: The peak value of the pseudo velocity V which is defined as $V = \omega_n \cdot D$, where ω_n is the natural frequency and D is the peak deformation of the same system. Pseudo velocity has units of velocity and it is related to the peak value of the strain energy E_{s0} stored in the system during the excitation, given by $E_{s0} = mV^2/2$.

- Pseudo acceleration: The peak value of the pseudo acceleration A which is defined as $a = \omega_n^2 \cdot D$, where ω_n is the natural frequency and D is the peak deformation of the same system. Pseudo acceleration has units of acceleration and it is related to the peak value of base shear V_{b0} or the peak value of the equivalent static force f_{s0} , given by $V_{b0} = f_{s0} = m \cdot A$.
- Relative Velocity: The peak value of the relative velocity.
- Acceleration: The peak value of the acceleration.

3.3.2 Input data

In order to input the data, select “Options” from the “Bouc Wen Analysis” menu. The following form will appear:

Figure 14: Input data for the generic Bouc-Wen model.

In the “General Data” frame, the following data is required:

- Mass: The mass of the system.

Both in the “X direction” and the “Y direction” frame, the following data is required:

- F_y : The yield force of the system.

- U_y : The yield displacement of the system.
- a : The ratio of post-yield to pre-yield (elastic) stiffness. Dimensionless.
- c : The viscous damping.
- Earthquake: In addition to or separately from the excitation, you can select the desired earthquake from the drop-down list box. Note that the modified form of the earthquake is used in the calculations.
- Excitation: In addition to or separately from the earthquake, you can select the desired excitation from the drop-down list box. Note that the modified form of the excitation is used in the calculations.
- Initial displacement: The initial displacement of the system.
- Initial velocity: The initial velocity of the system.

Note that the usage of an earthquake or an excitation is not compulsory; you can use an initial displacement or velocity instead to solve the free vibration problem.

Concluding Remarks

MySpec solves the free and forced vibration of SDOF and 2DOF systems for the linear and non-linear hysteretic behaviour that is common to a variety of structural and geotechnical problems. It reveals the main features of the hysteretic behaviour and introduces the user to the peculiarities of these systems in a didactic way.

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